**Mathematical Misconceptions in School Children**

**– Identification, Impact and Remediation with a Computer Based Assessment System.**

By Gordon Moore – March 2006

**Abstract**

The formation of mathematical concepts is a complex process and the evidence is that many pupils do not consistently experience success in developing correct conceptual structures and find difficulty in properly applying them to solving problems. The impact of some basic misconceptions of Year 9 pupils on examination performance is investigated and found to correlate strongly with the resulting “SATs level” obtained in the end of Key Stage 3 examination. It is suggested by the analysis of a student’s confidence in their answers that pupils are not always aware of having a misconception. Some basic feedback mechanisms to remediate misconceptions with computer based assessment are investigated and found to be unsuccessful in practice.

I confirm that the work in this Master’s Dissertation is wholly my own

Signed :

Date :

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Abstract

Acknowledgements

**1. Introduction**

1.1 Background

1.2 Purpose of Study and Research Questions

**2. Review of Literature**

2.1 Introduction

2.2 Concept and Schema – Formation and Use

2.2.1 Development of the Mathematical Mind

2.2.2 Concepts

2.2.3 Schema

2.2.4 Problem Solving

2.3 Misconception Types

2.4 Using CBA to Identify Misconceptions and Confidence Level

2.5 Using CBA to Remediate Misconceptions

**3. Methodology**

3.1 Introduction

3.2 Identifying Misconceptions using CBA

3.3 Ascertaining Confidence Levels

3.3.1 Ascertaining Confidence Levels Using the YAS package

3.3.2 Ascertaining Confidence by Interview

3.4 Effect of Different Feedback Strategies on Remediating Misconceptions

**4. Evaluation**

4.1 General Level of Misconceptions Found in Year 9

4.2 Analysis of Confidence Levels

4.3 Individual Question Analysis

4.4 Analysis of Pupil Transcripts

4.4.1 Recognition Factor

4.4.2 Level of Difficulty Identified

4.4.3 Anxiety Level

4.4.4 Emotional Attachment

4.4.5 Confusion Identified

4.4.6 Reasoning Processes and Explanation

4.4.7 Recognition of “Silly” Mistakes

4.4.8 Confidence

4.4.9 Mental “Spoonerism”

4.4.10 Summary

4.5 Analysis of Simple Feedback Strategies in Remediation of Misconceptions

**5. Conclusions**

**References**

**Appendices**

Appendix 1 Common misconceptions from the literature and practice.

Appendix 2 Questions and results from Year 7 pilot study.

Appendix 3 Questions and results from the Year 9 misconceptions and confidence study.

Appendix 4 Comparison of results for two Year 9 cohorts

Appendix 5 Pupil interview transcripts

Appendix 6 Basic pre and post test results for the second Year 9 cohort.

Appendix 7 Discussion of question types used in CBA

Appendix 8 Confidence rating per pupil (1st Cohort Y9) by pupil test score

**Chapter 1: Introduction**

* 1. **Background**

Most teachers of mathematics will have experienced pupils failing to properly develop a particular mathematical concept. This is usually identified when the pupil fails to solve a given problem correctly, or when participating in some verbal exchange. It is especially disconcerting when much effort and work has been invested in the teaching and learning process.

It has been postulated that this is due to a failure in the pupils’ memory processes, especially between tuition and test. However, some research showed that this is unlikely (Moore 2001, 2002). What then, can be behind the cognitive failure to answer seemingly simple questions and solve fairly innocuous problems?

A possible answer to this was evidenced in the author’s own practice when noticing how often similar misconceptions arose when marking. For example responses such as: 3.7 × 10 = 3.70, x2 is the same as 2x and 3(x+4) = 3x+4 are often seen.

Notes were made over time of these misconceptions and a wide range of them were recorded – see appendix 1. The obvious questions were why and how did these errors arise, what impact do they have upon performance and progress and what can be done about them.

* 1. **Purpose of Study and Research Questions**

In order to answer these questions it is useful to investigate the cognitive processes that might occur during the development of mathematical concepts and the learning of simple problem solving procedures. It is important to determine if all misconceptions arise from similar processes or do misconceptions have a varied etymology.

Clearly even a single misconception can cause damage to progress and attainment, but what if a pupil has a variety of misconceptions in different areas of mathematics. It would seem reasonable to expect that the more misconceptions a pupil has then the more likely the pupil is to experience failure in examinations. This dissertation will establish the validity of this hypothesis.

It is important to identify what misconceptions a pupil has so that corrective action may be taken. In the past this would involve a teacher manually analysing test results, perhaps using a grid, so that individual and class misconceptions could be identified. This process is very involved and few teachers would find the time to do it. Computer Based Assessment (CBA) and appropriate software can provide feedback immediately which can then be analysed for evidence of misconceptions in various ways: for both individual pupils and by class. It is also possible to use such software to identify if a pupil is aware of having a particular misconception by recording how confident they are of a selected or given answer. This research will investigate how effective this use of CBA is in practice.

In the past correcting identified misconceptions would fall to the teacher, but generally personal attention would usually be limited and untimely. If CBA can detect a misconception, can it then be used to remediate the misconception? Some basic feedback strategies are investigated to determine how effective they are in this role.

So the basic research questions this study addresses are:

* How do misconceptions arise and what is their nature.
* Is there a correlation between the level of misconceptions in a Year 9 pupil and their performance in the end of key stage three examination? Therefore, can such a test be a good predictor of examination performance?
* Does a pupil’s confidence level in answering a question correlate with selecting correct answers, i.e. are school pupils aware of their misconceptions?
* A comparison into the effectiveness of remediating misconceptions using simple feedback strategies with CBA.

**Chapter 2. Review of the Literature.**

**2.1 Introduction**

The literature on misconceptions, cognitive development and on computer based assessment/computer aided assessment is vast (Smith, diSessa and Roschelle 1993) and by necessity only a limited amount can be surveyed.

[Note: This research does not consider explicitly the issue of dyscalculia whereby a student who is generally able experiences a specific, severe difficulty with a mathematical skill or ability (Price and Youe 2000).]

The first key issue is to look at how concepts develop in the mind and to try and identify where and how they might become misconceptions.

**2.2 Concept and Schema – Formation and Use**

**2.2.1 Development of the mathematical mind.**

In the 1940s and 50s Piaget studied the development of children’s mathematical thinking extensively. He suggested that a child starts with a clean cognitive slate – tabula rasa (Smith, diSessa and Roschelle 1993) – and that as a result of experience, mental structures are constructed that enables the child to make sense of the world and to engage with it in a meaningful way. He postulated four basic stages in development: sensori-motor, pre-operational, concrete operational and formal operational (Copeland 1979 p20-25). These are well known. (Interestingly Skemp (1979) and Tall (2004) have suggested a fifth stage of being able to work with abstractions at an even higher level as in advanced mathematics.)

However the tabula rasa view would seem to be mistaken. Babies have an innate ability to distinguish between one, two or three objects and to be aware of change in the number of objects. They seem able to realise that 1 + 1 = 2 and that 2 – 1 = 1 (Devlin 2000 p28).

However, apart from some fundamental abilities, Piaget’s view of developmental stages seems sound, though it is perhaps more likely a continuum of change with some abilities developing at different rates and to different levels.

Adhami (2002) notes that there is a “tentative match” between the range of Piagetian thinking levels and the level descriptors (1 to 8) of the National Curriculum for Mathematics in England. Figure 2.2.1.1 clearly shows the increase in mathematical understanding over time.

Such development requires the growth of mental cognitive structures. There are an astonishing number of theories explaining how this might occur and the structures involved (see the Theory into Practice database – <http://tip.psychology.org/index.html>). In this review the theoretical ideas of the late Professor Richard Skemp will mainly be considered (Skemp 1962a, 1962b, 1962c, 1976, 1979, 1979b, 1986). His basic idea correlates with Piaget’s of the development of a theoretical structure called a *schema* composed of interconnecting *concept* structures. In order to achieve a particular goal (i.e. solve a maths problem) the mind must use or create a plan that takes the person from the present state to the goal state. This is achieved by means of *director systems*. As this occurs the organism receives signals indicating the current status of the plan. Generally, for the plan to succeed the conceptual structures must be accurate.

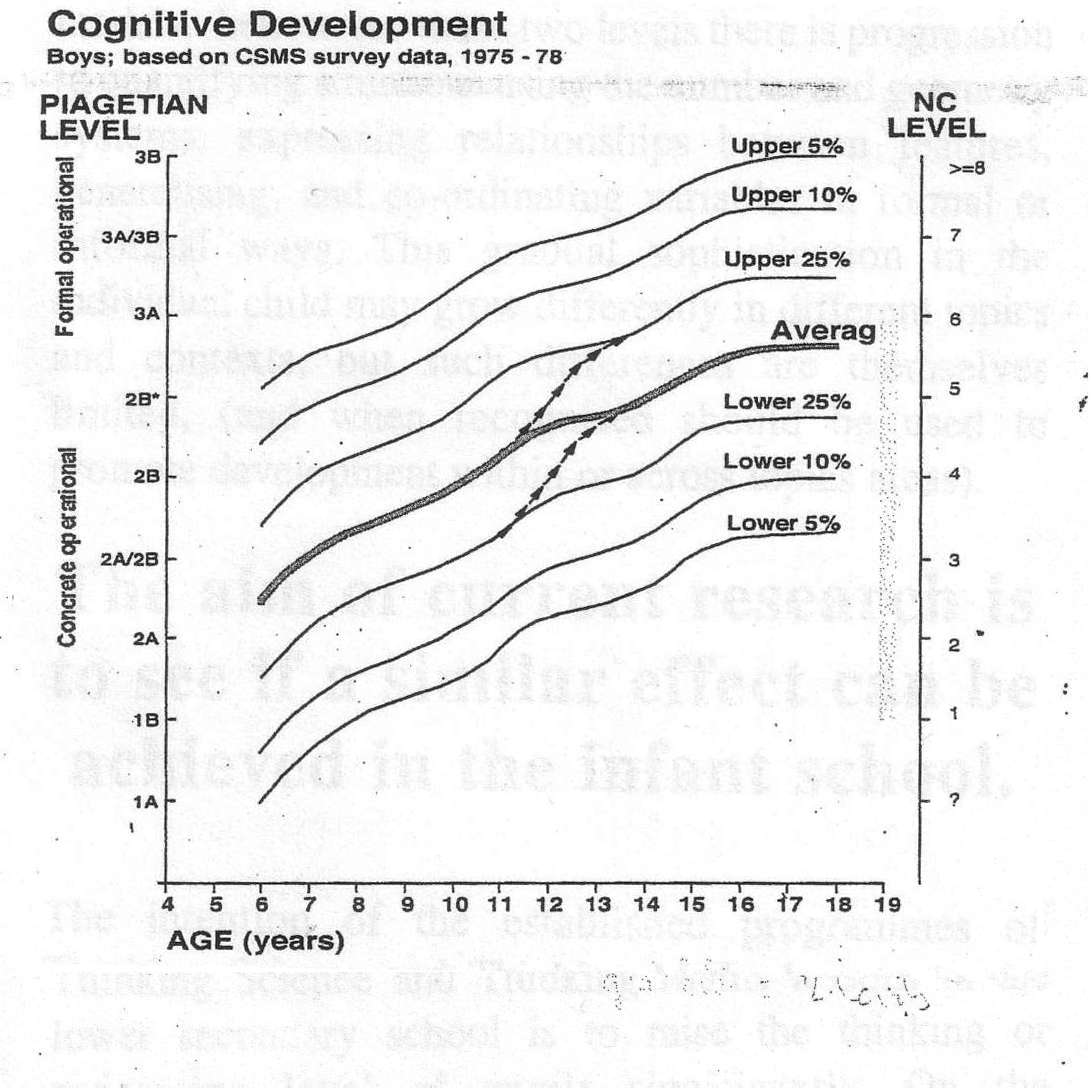


Figure 2.2.1.1 Cognitive Development (From Concepts in Secondary Mathematics and Science, showing data from 14,000 pupils given a battery of Piagetian tasks.)

**2.2.2 Concepts**

The way that conceptual structures develop is by the mind abstracting the characteristics that identify an object as having some particular quality (Skemp 1979b p116, Eysenck 1993 p152). For example the concept of redness is obtained by looking at various red objects (and not-red objects) and in some manner identifying the characteristic of redness. (This mechanism is undefined in Skemp’s theory.)

It was a fundamental point with Skemp, that in order to develop good concepts, good examples of the concept were required, otherwise all that are obtained are facts. It is helpful to concept formation if the examples, initially at least, have low “noise” (Skemp 1979b p123, Sfard 1990), i.e. the characteristic that is being extracted is not muffled or hidden by irrelevant information. Non-examples also help highlight the feature extraction process. It is useful if the examples are spatially and/or temporally close together (Skemp 1979b p121). Once, and if, a concept is formed then a meaningful definition of it may be provided (Tall & Vinner 1981). It has been noted that some teaching practice works the opposite way round, by providing a definition first and then trying to use it with examples (Stacey and MacGregor from Tall and Thomas 2002 p221, White and Mitchelmore *ibid* p235-6, 238).

In general it has been seen that concepts are stable and resistant to change (Skemp 1979b p128). This is a good thing because we don’t want to have our concepts continually updated on the basis of every experience we have, but it is a bad thing if the concept is malformed, because there will be difficulty in changing it.

Difficulties can arise in concept formation if the examples or models are not chosen carefully. Fishbein (1977) noted that graphs are powerful tools, but can be misread, e.g. a travel graph is not about hills! Mokros and Russell (1995) commented about short-circuits occurring, for instance when the algorithm for calculating the mean is introduced before pupils have gained understanding about representativeness. Clearly, practice (lots of examples) and repetition is helpful in concept formation (Skemp 1979b p116, Eysenck 1993 p53,58, Anderson, Reder and Simon 2000).

Cognitive conflict (Tall and Vinner 1981) can also cause difficulties (as well as in developing new areas of mathematics). One example is the development of the concept of negative numbers by using a question like 5 – 9. This development needs care, as evidenced historically by the unease, even horror that earlier mathematicians regarded the integers – the numeri absurdi of Stifel (1487-1567) or the numeri ficti of Cardano (1501-1576).

A very powerful tool used to label, identify and work with a concept is the use of a symbol (Skemp 1986 p79,82, Tall & Vinner 1981, Skemp 1962a, Gray from Tall and Thomas 2002 p205-6). However it is unfortunate in mathematics that not all symbols are uniquely defined. For example x-1 means 1/x, yet sin-1 refers to the inverse sine. Another example at a more elementary level is the use of the **−** symbol. It can mean subtraction or it can be used as a symbol for a negative quantity. Students can easily miss the subtlety and sometime lose sight of what the marks or squiggles represent (Davis and Tall from Tall and Thomas 2000 p138).

**2.2.3 Schema**

Independent concepts are only useful up to a point. They can only identify an object as being a example of the concept or not. Concepts would appear to become interconnected in a mesh like structure called a schema. For instance the concept of furniture has an “interiority” of concepts such as chair, table etc (Skemp 1979b p114, 115, q.v. the “concept image” of Tall and Vinner 1981).

Schemas are created by incorporating concepts into its structure by the processes of assimilation and accommodation (or re-representation, reconstruction) (Anderson, Reder and Simon 2000, Skemp 1962b, 1979b).

Assimilation occurs essentially by the addition of the new concept structure. Accommodation on the other hand occurs as a result of a conflict between the existing structure and the new one. For example, the development of working with integers or the discovery that multiplying doesn’t always make bigger. Since schemas are also stable and resistant to change this process can be difficult, sometimes impossible, for an individual. It is not simply a case of replacing a link or re-writing a concept/schema. The existing concept/schema may well be valid in a limited context and may also be usefully linked in with other schema as well.

Accommodation may be difficult, if not impossible, if the underlying schema is already faulty.

If a new concept is successfully incorporated into the existing structure then we say we “understand”. Often though we may delude ourselves by thinking we understand, when we do not, typified by Vinner (1987) who referred to it as the “nodding syndrome”. We nod our agreement to be polite.

**2.2.4 Problem Solving – using the concept/schema.**

How can it be known if a concept or schema has developed correctly? As far as Skemp was concerned it was by *what a person does* in a situation which requires the use of the concept. (This should include recognising presented examples of a concept!)

Skemp’s theory suggests that a concept is activated in the mind when an example of it is encountered. For instance some item of information in a question may act as a cue and trigger the cognitive response (Oliver 1989). This is usually an uncontrolled set of associations (Vinner 1997). As one concept is activated it may cause other concepts to become activated to some extent. A person may become aware of these if the level of activation achieves a certain threshold level (Collins and Loftus theory of spreading activation – Eysenck 1993 p85, Anderson 2000 p183, 222). The concepts most closely connected will become more excited than those distantly related (Skemp 1979b p131).

The essence of problem solving is to work with the most appropriate of the activated structures. Of course inappropriate concepts may become activated, possibly due to (incorrect?) prior learning, yet the pupil may not be able to discern this or discriminate between them. Alternatively, different, possibly conflicting schema may become activated for a single event leading to confusion (Tall and Vinner 1981).

For example “pattern interference” in the question may cause errors (Anderson, Reder and Simon 2000, Devlin 2000 p62, Vinner 1997). Here is an example from Vinner:

*“The milkman bought on Monday 7 bottles of milk. That was 4 bottles less than on Sunday. How many bottles were bought on Sunday?”*

Of course the answer is 7 + 4 = 11, but the verbal cue of “fewer” causes some children to think of subtraction and do 7 – 4 = 3.

The actual mechanisms involved in problem solving are necessarily complex and Skemp’s theory devotes much space to it. For our purposes we accept that procedural knowledge or plans, i.e. the techniques and algorithms used to solve problems, when they are identified, must be integrated into the cognitive structures in some way, possibly as specialised schema. The director systems used to solve problems, i.e. take the organism to the goal state, must be able to access these, or to create new plans using the existing schema. It is useful to note however that some plans are already hard-wired in as instincts and that some become “habits”. Of course good “habits” must be set up carefully (Skemp 1979b p168). A further development of these useful plans and with an associated symbol leads to the idea of a procept – an amalgam of process and concept. For example the plus symbol + (Gray and Tall 1994). This is either the counting on process or the number facts of addition. Procepts reduce the cognitive load and provides the problem solver with more flexible options.

If a ready made plan does not exist, then problem solvers have real difficulties. The student could use a plan that is inappropriate to the problem. Hall (2002) referred to these as mal-rules, which seem to work, but are only valid under restricted situations (Smit, Oosterhout and Wolff 1996, Zehavi 1997). For example using the arithmetic mean when the median might be more appropriate. For many pupils it might be that the mean is the only technique they have at their disposal. Similarly using the wrong calculation for a weighted mean (Pollatsek, Lima and Well 1981).

Sometimes the student may not be able to see how the posed problem relates to existing knowledge. Wertheimer is known for researching how well students could work out the area of a parallelogram when the orientation of the parallelogram was changed from the way it was usually taught (Beaumont 1960). Fishbein and Muzicant (Tall and Thomas 2003 p52) discussed research in which 90% of 15-16 year olds knew the definition of a parallelogram, but only 68% could actually recognise one from an assortment of figures. Van Heile retorted to a child when asked if a triangle was isosceles when lying on one of its equal sides: “Is a dog still a dog when lying on its back!” (Tall and Thomas 2002 p30).

The ability to reflect is a key component of successful problem solving both in considering the answer obtained and in considering alternative strategies to solve a problem. This facility is not often well developed in pupils many of whom are still at a concrete operational stage until 16 or even older (see figure 2.2.1.1).

It is noted that many pupils don’t even like to reflect (Van Heile from Tall and Thomas p43) and generally they feel they get along quite well without it (Dienes from Tall and Thomas 2002 p23-4).

Linked to this are Vinner’s ideas on pseudo-cognitive thinking (Vinner 1997). An example of this is adapted from his paper. When faced with a question the pupil is faced with a conflict. They may not want to actually initiate the cognitive demands involved in properly solving the problem, but are in a social environment which demands some activity. For instance the child is required to solve a problem involving finding the perimeter of a rectangle of sides 7cm and 4cm.

The correct cognitive process involves identifying the word rectangle and perimeter, accessing the required conceptual structure and initiating and carrying out the plan to add the two sides and double, or double each side and add or even just to recognise that opposite sides are equal and total the sides.

However this involves intellectual processing demands and the student would like an easier life. They might suppose that since there are two numbers they could simply add them – and some do, or being a little more sophisticated they may think adding is trivial, so perhaps they should do something more “mathematical” such as multiplying. They get an answer and are content and have satisfied the social necessity to do some work.

Even more disturbing is that of getting the correct answer by a wrong process, for example a rectangle with sides 3cm and 6cm. In this case any misconception might not be identified (q.v. Fishbein and Muzicant in Tall and Thomas 2002 p53, Gardner-Medwin 2004, Oliver 1989, Borgen and Manu 2002).

Cooper and Dunne (2000) in a comprehensive review of the literature and from their own research also identify the issue of “realistic” test items. It would appear that many pupils, in particular of lower ability and lower social class, have a propensity to consider not the hypothetical situation described, but their own actual experience – this is called differential item functioning. Also, it is not always easy for pupils to ascertain the mathematical requirements from the “noise” of the problem (Clausen-May 2001 p29-30). However some commentators contradict this view feeling that situational representations facilitate mathematical problem solving, often referring to the research on Brazilian street children by Caraher, Caraher and Schliemann (in Stern and Mevarech 1996 for instance).

There are of course many more factors to consider and details to attend to in such a complex process, but the above discussion should give at least a starting outline for working with.

**2.3 Misconception Types and Test Design**

Given the above, it can be seen that misconceptions are not only varied, but may have very different causes. In brief we might attempt to determine some such causes:

* The examples used in developing a concept were inadequate for the task. This could be due to a failure to provide adequate practice, or to provide appropriate examples. Perhaps some concepts are seen as “obvious”.
* The pupil is not cognitively developed enough to comprehend the concept, or there is some physical brain malfunction or damage. (This would be an interesting question to research especially considering for instance the wide range of disorders associated with autism for instance.)
* A prior, wrong concept already exists and there is a failure to change or overwrite it.
* A particular model used for developing a concept is taken beyond its useful environment.
* A cognitive conflict develops which is not fully attended to.
* A symbol is misinterpreted or used inappropriately
* The concept is not added properly to an existing schema
* The existing schema fails to accommodate to additional conceptual knowledge.
* The existing schema incorrectly accommodates the new information.
* The existing schema is faulty.
* The schema is used outside its valid context.
* Problem solving mechanisms use old, faulty schema rather than the new updated ones.
* Many possible schema are available and the wrong one(s) is(are) selected.
* New concepts/schema interfere with other correct concepts/schema and cause confusion.
* There is pattern interference, situational, or presentational effects in the problem.
* A correct schema is not available, so an incorrect one is deliberately selected instead.
* Failure to reflect and alter the problem solving plan.
* Pseudo-cognitive activity.
* Affectional considerations

Much research into specific misconceptions identifies many different causes and aspects of the misconception. For example in the topic of mensuration Outhred and Mitchelmore (2000) identified that some very basic understanding of grids or arrays was pre-requisite for a fuller understanding of area, whereas many students were simply concerned with just knowing formulae (Baturo and Nason 1996). Similarly Mokros and Russell (1995) noted that many saw the mean as just a calculation and that few saw the underlying concept of a balance point or even of sharing (q.v. Meyer & Channell 1995).

Given this, it seems to be a huge task to try and categorise misconceptions by type, since any particular misconception could have a variety of causes and failure at ever lower levels of the concept structure. Misconceptions seem to be specifically tied to the actual concept.

For instance, consider 72 being manipulated as 7 x 2. Is the misconception because of misreading the superscript 2 and seeing 7 x 2. Is the cause a failure to know about indices? Is the failure simply laziness and multiplying by 2 is easier than multiplying by itself and of course 7 x 7 is hard, or is the number even seen as 72?

If the topic of indices has been taught, why has the misconception arisen. Does the correct understanding actually exist, but is not being accessed and is a posed problem perhaps being solved in a pseudo-cognitive way?

Rather than categorising misconceptions, what might be more useful is to carry out analyses of the various misconceptions that arise and try to develop questions and options that can identify these factors.

A notable example of this is the work done by Neill (2000) in New Zealand whereby a National Assessment Resource Bank is used by teachers nationally and results can then be analysed to try and uncover mistaken thinking and misconceptions that occur. Analysis of user input provides valuable diagnostic information. Neill points out that usually less evidence is obtainable from the use of multi-choice answers. The system is designed to provide tests, but as a by-product provides diagnostic information.

The approach taken in this research then, is to try and directly uncover specific misconceptions and so common misconceptions identified in the literature and in the author’s own practice were used to create the test instruments.

**2.4 Using CBA to Identify Misconceptions and in Assessing Confidence Levels**

Computer based Assessment systems although widespread in Higher Education are less commonly found in schools. However more and more schools are starting to use either bespoke software or packages such as Virtual Learning Environments which incorporate CBA.

Essentially a CBA package will offer a Test/Question Creator module, a module for setting up a list of participants (test takers), a Test Assignment module and the actual Delivery application itself whereby a participant takes a test. This could be by a Windows™ program or more commonly using a web based browser. A final set of applications should provide various levels of analysis of the data.

For the purposes of identifying misconceptions and remediating them, the CBA system should offer a variety of question types and some kind of feedback mechanism.

There are a wide variety of question types that have been considered in the literature and each offer various advantages and disadvantages. The main types are discussed in Appendix 7. However for the purposes of this study the question type is practically limited to multiple choice. In this a question stem is presented to the participant and a variety of options is made available from which the participant chooses one option.

For the purposes of identifying misconceptions this has the advantage that the distractors can be carefully chosen so as to perhaps reveal the kind of wrong thinking the pupil is engaged in, though in practice it can be difficult to come up with sensible options.

In particular, improper reasoning, pseudo-conceptual thinking or just being plain lucky may select the correct response, even though there is an actual misconception in the pupil’s mind (Lawson 1999, Kupermintz, Le and Snow 1999). Sometimes unanticipated language clues may guide the participant to the correct answer (Neill 2000, Lawson 1999), in particular by using intelligent guessing (Harper 2003). This does not allow deep seated misunderstandings to be revealed.

Usually one question would be asked for a particular concept, but it might be better, given the above considerations, if a number of questions were asked on a topic. A misconception will often reveal itself in the same way under similar conditions, whereas a slip is sporadic.

Another issue affecting assessment validity is question ambiguity (Hodson, Saunders and Stubbs 2002), and it is advisable that pre-testing and checking of the questions is carried out. Hawkes (1998) notes that students are often quick to identify such issues!

The reliability of the test instrument also needs to be considered. A particular question should reveal the same misconception for many participants and the same participant if it were possible to repeat the test. It might be useful to present the same type of question in a variety of formats.

However, there is a problem here of overloading the participant with too many questions so that the test becomes onerous or boring and the danger that consequently the test taker doesn’t take as much care. It is recommended that tests should not last longer than 90 minutes or tiredness becomes an issue (Oliver 2000, Twomey, Nichol and Smart 1999, Clariana and Wallace 2002, Bull and McKenna 2001 p51) and should have about 40 questions an hour as a maximum (Bull and McKenna 2001 p51). For school pupils these limits should probably be less.

Another advantage of CBA is that software can be written that allows additional information to be recorded from the participant. In this study the ideas of Confidence Based Marking (CBM) were used to provide a way of recording the confidence of the test taker as they answered a question.

Some researchers (Gardener-Medwin 2004, Gardener-Medwin & Gahan 2003) have looked at doing this by asking the student to provide a level of confidence for their selected response to the question itself. The student can select from three levels of confidence (1= low, 3=high). These levels are then used to compute a score based upon their correct or incorrect response. Correct answers with a high degree of confidence are rewarded, but wrong answers with a high degree of confidence are punished. The score is returned after each question and the user can see how well their strategy is working. This technique causes the participant to think more about the answer they are choosing, relating it to their degree of belief as to how well they think they understand the question and the degree to which they feel their answer is correct. Apparently students find the process quite intuitive and having a few goes soon provides them with the idea more than explanations.

A potential drawback is perhaps if students have misconceptions that aren’t recognised by themselves as such. Also there may be an element of game-playing strategy involved in selecting confidence levels - playing safe.

Another approach taken by Petr (as reported in Lo, Wang and Yeh, 2003) is to get the user to rate the degree of confidence for each option available. This does require a lot of user interactions not only physically, but mentally in determining a response to each option. This method has clear attractions for use in multi-response situations such as in deciding the appropriateness of a preposition in an English sentence. In such a situation more than one answer may be appropriate, but one may be better or more idiomatic than another. These researchers noted that the students made significant improvements when answering using confidence scores as opposed to simply selecting an option. The system was forcing them to consider and evaluate each response.

For the purposes of this research the ideas of actually scoring by Confidence Based Marking ideas were rejected as being inappropriate to determining misconceptions. Clearly such systems have a role to play in helping pupils to improve their performance and in overcoming misconceptions, but the time involved in training precluded the use in this research.

However it was decided to record how confident a pupil felt about their response as it was felt that this kind of signalling would be helpful in identifying if the pupil did indeed have a misconception. Skemp’s theory suggested that feeling of unease and disquiet should be generated if a misconception or misunderstanding was encountered or used.

**2.5 Using CBA to Remediate Misconceptions**

It may be assumed naively that once a misconception is identified then it is simply a matter of replacing the wrong concept with a correct one. However the issue is not so straightforward.

Much misconception research seems to be focussed on one particular error suggesting that the fault can be easily rectified in isolation (Smith, diSessa & Roschelle 1993). One method of remediation is by “explaining”. Stacey & MacGregor (from Tall & Thomas 2002 p228) report Skemp’s views on this (Skemp 1977 p 76):

* *The wrong schema may be in use – so explaining simply activates the right schema*
* *The gap between the new idea and the existing schema is too great – so supply intervening steps*

White & Mitchelmore (in Tall and Thomas 2002 p 250) noted though that additional tuition does not always result in success. For example the 40 students in their study studied a 24-hour course intended to make the concept of rate of change more meaningful – but the only detectable result was an increase in the number of errors in symbolising a derivative!

It is suggested by some that simple telling, repetition of the information or even improving the clarity of the explanation may not help (Mestre 1989, Smit, Oosterhout & Wolff 1996). Students are often emotionally attached to the misconceptions they have (Mestre 1989). From a constructivist point of view, telling rarely works anyway. What is needed is discussion, communication, reflection and negotiation (Smith, diSessa & Roschelle 1993). Obviously this is difficult for a CBA system.

As has been discussed, conceptual structures are stable and may actually be relevant to limited contexts or valid when used within some particular schema. If a concept is invalid under certain circumstances it may be possible to simply explain that in the given situation additional considerations must be examined. Alternatively, pointing out to the student at the time of (re)learning a concept how far they can go with it may help. It may also help to deliberately point out errors that may arise.

Pupils need to be made aware of the dangers of over generalisation of concepts and that this may almost inevitably lead to misconceptions (Olivier 1989). However earlier ideas may be so firmly entrenched that it is difficult to stop their [unconscious] effect. Even if a misconception seems to be overcome, the student may return to it later (Mestre 1989). For example, Hall (2002) carried out research with some students involved in simplifying algebraic fractions. Even after instruction on remediating a particular error, students quite often fell back into the old mistaken method.

This is perhaps related to the issue of permanence of memory traces. For example Noice and Noice (1997) noted that actors find it easier to learn the lines of a play if it is one they have learnt before, even if that was many decades previously.

However in other circumstances the nature of the misconception is that the conceptual structure must be reconstructed (i.e. accommodated) (Mestre 1989). As has been seen this is neither easy nor without cost. Smith, diSessa, Roschelle (1993) assert, however, that not all misconceptions are resistant to change. Appropriate interventions can result in rapid and deep conceptual change in a short period of time (though no example is provided).

A technique that might help in reconstruction is by the use of “conflict teaching” where inconsistencies are presented to the learner so that he or she can see the need to make changes to their mental structures and hopefully do so (Swedosh 1999).

This view is strongly argued against by the constructivist position of Smith, diSessa & Roschelle (1993) who express the thought that confrontation is a denial of the validity of a student’s ideas; can it be simply said that a misconception is a mistake, is it not merely “unproductive”? This seems a little obtuse though. The impact of replacement causing detrimental psychological stress to the student has to be considered; it may “drive them underground”.

Another idea is to get the student to focus on the emotional signalling generated whilst they are attempting a problem. By this a student may learn to reflect more on how well they are actually dealing with the problem and be able to take more appropriate action, even if it is simply to realise and accept that they actually have a problem. The original aims of this research included looking at this aspect, but it leads to a wider focus than appropriate in this dissertation. The pupil interviews provide some evidence though that this is difficult.

In order to change a misconception the student must firstly become aware that they have it. A feedback mechanism must provide this indication to a student.

In a CBA system, this could take place as soon as the student has finally decided on an answer, usually by the fact they are moving on to another question (although this stops the pupil being allowed to reconsider answers at the end of a test – review) or at the end of a test when the pupil can either see what score they have and possibly see which questions they got wrong. This could be by being presented with a list of questions indicating which are wrong or by allowing the pupils to scroll through the questions answered and which shows which are correct and which are wrong.

Different levels of feedback can be offered. This could simply be a score, or an indication of which questions they got wrong which might allow them an opportunity to mentally try the question again. The CBA mechanism might actually allow them to retry the question, perhaps even recording their attempts. This may be once or many times. CBA may allow the pupil to retry or retake the test, perhaps frequently (TAL, Buchanan 2000). The advantage of CBA here is obvious: computers can provide immediate feedback, never get bored and are patient (Pellone 1991, Buchanan 2000 ).

If more detailed feedback is to be provided then the nature of that feedback needs to be considered and it needs to be determined what it is intended to do to the misconception.

It can be seen that the level of detail provided in the feedback mechanism might be brief, or may provide a reference to further material providing explanations (Buchanan 2000) or may even cause the CBA mechanism to jump to a sophisticated web page providing much detail or even to a learning system which attempts to re-teach the concept (Hodson, Saunders & Stubbs 2002, Nguyen-Xuan, Nicaud & Gellis 1997).

It is suggested that the feedback or learning material provided should be difficult enough to influence the student’s thinking. The material should ideally be interactive as opposed to passive such as is often found, and if possible should be adaptive to the learner’s input and prior learning. Interactivity should relate to an actual interaction with a learner’s thought processes rather than simply a dynamic web page or animation (Ketamo & Multisilta 2003). Lo, Wang and Yeh (2003) point out that that there can be dangers in cognitively overloading the participant if too much is required from them, or if too much is offered.

Feedback may require time for it to become effective. The user might just glance at the material, but what is really required is time for reflection and consideration (Buchanan 2000). It is also suggested that feedback should not introduce new concepts (Smit, Oosterhout & Wolff 1996). Small changes in the feedback can lead to significant changes in learning (Nguyen-Xuan, Nicaud & Gellis 1997) so, feedback should be carefully designed and tested.

However, the time and effort required to design and implement sophisticated feedback mechanisms is considerable and outside the constraints a teacher has. The question remains as to whether simple feedback can play any part in promoting change in a pupil’s conceptions and this research will investigate this.

**Chapter 3. Methodology**

**3.1 Introduction**

In order to answer the research questions identified in 1.2 it was decided to use a software package called YAS (Your Assessment System). YAS has been designed (by the author) to allow the creation and presentation of test questions and to store responses in an Access database allowing for easy analysis. Another advantage is that YAS may be reprogrammed as required to allow for the use of more sophisticated techniques such as recording confidence levels. The commercial software package Perception (by Questionmark) had been considered and indeed purchased by the school, but in practice had proved to be unwieldy and inflexible in use.

Questions are presented to the test taker (participant) using a standard web based interface. There are some inbuilt analysis applications, but data can easily be exported to an Excel spreadsheet for more sophisticated analysis. Special application programs (written in the C# language under Microsoft .Net) allow for the creation and editing of test options and for scheduling tests. More details and screen shots can be found at [www.sandcastle.me.uk](http://www.sandcastle.me.uk) .

It was decided that all questions would take the form of a multiple choice format. YAS does allow for text entry, but the routines for parsing unusual responses are not implemented.

All questions did allow for the selection of a “Don’t Know” or “Can’t Do” type response where appropriate, as it was hoped that this might allow for additional analysis and reduce guessing effects. As it turned out few pupils used this option.

**Caution**

The tests were administered in a normal school ICT laboratory, either by the author, but more usually by the Head of ICT when she had the classes being used in this research in her normal timetable. Pupils are quite used to taking CBA tests and no special instruction had to be provided for this research. However a special instruction sheet was provided to the head of ICT which detailed exactly what the purpose of the tests was, a script to be read to the pupils about the selection of the confidence option and to allow discussion on this. Pupils seemed quite happy about this and no problems were experienced by its inclusion.

No special invigilation practices were carried out, although pupils were monitored and exhorted to keep their eyes on their own screen. However it cannot be guaranteed that covert scanning of another screen did not take place as pupils do sit close to each other. Also the author was not able to be on hand at every test and a particular test had to take place over a period of a few days as the pupils had their ICT class. Correspondingly there is a danger that the results in this dissertation may be unreliable and should be treated with caution, though there is no evidence of any wrong action on the part of the “invigilator” or of the pupils.

**3.2 Identifying Misconceptions using CBA**

The initial research was concerned with identifying the level of misconceptions a pupil has and in then determining the effect of this upon exam performance.

It was decided to construct a 20 item test instrument which would be administered to Year 7 pupils as a pilot study to gain experience in this type of test and to iron out any problems before the test was administered to all of Year 9.

Although it might have been preferable to have a more formal selection of test items covering specific areas of misconception, it was decided to try out a wide range of misconceptions from various strands of the Key Stage 3 specification and to see what resulted from these. The questions were constructed from examples found in the literature and from the author’s own practice and experience and were held to be fairly representative of common misconceptions evidenced in school children. One question was prompted by a colleague’s request, namely the question on angle size and relative scale of the image.

The questions were sequenced in the same order to each pupil, though the software allows for the randomisation of question presentation if required.

The pilot was administered to all the pupils in my own Year 7 class and a colleague in ICT administered the test to any other Year 7 classes she taught. No pupils from the bottom set were tested.

The questions and results for the pilot are attached in Appendix 2.

As a result of the pilot study it was clear that one question did not have the correct response identified and that some questions were at an inappropriate level for the pupils, e.g. the question on standard form, which is a level 8 question. Some questions it was felt did not really provide any gainful knowledge, e.g. What is the correct way to say 3.14, and a question on time was complex to decipher. It was also felt that there needed to be more questions.

As a result a new 30 question test was constructed. In particular more emphasis was given to questions on place values, calculation, fractions and negative numbers as it was clear that this area was identifying some major issues. However it was still felt important to cover the four basic strands of the English National Curriculum – Number, Algebra, Shape and Space and Handling Data. However many questions had to be excluded in order to keep the test to a manageable quantity. The questions were again selected mainly on the basis of the author’s own judgement and preference rather than according to any particular theoretical consideration, though of course the literature did influence this selection and some are directly adapted from the these – see Appendix 1.

The misconceptions that were chosen are:

1) What is the correct answer to 0.7 × 10? (7.0\*, 0.70, 70, 0.07)

2) What is the correct answer to 0.7 ÷ 0.1? (0.07, 0.70, 7.0\*, 70)   
3) What is the value of 1 - 0.07? (0.03, 1.07, 0.93\*, 1.03)   
4) What is -3 × -2 ? (-6, -5, +6\*, +5)   
5) What is -3 - -2 ? (-1\*, -5, +5, +1)   
6) Which of the following is the same as 3/5? (0.35, 0.3, 0.53, 0.6\*)   
7) Which of the following is the same as 0.35 ? (35/10, 3/5, 5,3. 35/100\*)   
8) What is the value of 0.1 × 0.1? (0.1, 0.01\*, 10, 0.2)

9) Which is larger, 0.28 or 0.9 ? (0.28, 0.9\*, both the same, depends)   
10) What is the answer to this calculation: 31-17 (column) (26, 14\*, 16, 24)

11) A farmer has twelve cows. All but five die. How many cows does the farmer now have? (12, 7, 5\*, 0)   
12) Which of the following numbers is between 2.5 and 2.6? (2.51/2, 2.7, 2.505\*, 2.65)

13) What is the answer to 2 divided by 8 ? (1/4\*, 4, 2.8, Can’t be done)

14) What is the answer to 3/5 + 1/10 ? (7/10\*, 4/15, 4/5, 3/50)

15) Which of the following is the same as 7%? (0.7, 7, 1/7, 0.07\*)  
16) If x = 3 what is the value of 2x2 ? (12, 36, 529, 18\*)   
17) What is the answer to p2 × p3 ? (p5 \*, p6, p23, 2p5)  
18) Simplify the following expression 3(x + 4)? (3x+4, 3x+34, 3x+12\*, it depends on x)  
19) Which of the options is the same as (a + b)2? (a2 + 2ab + b2\*, a2 + b2, 2a + 2b, 2ab2, can’t be done)

20) What is the perimeter of this rectangle (7 by 5)? (35, 75, 12, 24\*)   
21) What is the area of this triangle (h=5, b=6 right)? (11, 30, 22, 15\*)

22) Which of these two angles is larger? (same angle, but A drawn smaller) (A, B, same\*, impossible to know)

23) What is the correct name for this shape (Pentagon) ? (Quadrilateral, Pentagon\*, Hexagon, Octagon)

24) What is the correct formula for the circumference of a circle? (π × r2, π × r, π × 2r2, π × 2r\*)

25) What is the median of the set of numbers : 3, 7, 9, 2, 2? (2, 3\*, 7, 9)  
26) A fair coin is tossed 9 times. It comes up heads every time. Which of the following statements is true.

(The coin must come up tails next because it is a fair coin, This coin is not a fair coin, otherwise tails would have come up more times, On the next throw a tail is more likely than a head, There is an evens chance of heads coming up again\*)

27) I catch 3 fish, my two friends catch 2 fish each. What is the mean number of fish we catch?

(divided by 3, i.e. 1.66666..., 7 divided by 3, i.e. 2.33333...\*, 5 divided by 2 i.e. 2.5, 7 divided by 2, i.e.

3.5)  
28) A taxi can take 4 people. How many taxis are needed to take 18 people to the theatre? (3, 4, 5\*, 9)  
29) You are told that a number x is > -2. Which of the following is true? (x could be -1\*, x could be -2, x could

be -3, any number)  
30) What is the answer to 3 + 4 × 2? (9, 11\*, 14, 16)

It is possible to group the questions into a number of areas:

* Q3, 9, 10 and 12 deal with number size, order and relationship inc subtraction.
* Q1, 2, 8, 13, 28 and 30 deal with calculation, in particular place value.
* Q4, 5 are concerned with the integers
* Q6, 7, 14, 15 work with the relationship between fractions, decimals and percentages.
* Q16, 17, 18,19 and 29 are from algebra topics.
* Q20, 21, 22, 23, 24 deal with shape and mensuration.
* Q25, 27 are related to handling data
* Q26 is a probability question
* Q11 is a pattern interference question.

It was also felt that, although the test had to be accessible to all Year 9 pupils, including those at the bottom end of the ability range, it was important to be able to discriminate at the top end as well. Accordingly some harder algebra questions were included.

The main test was administered to all Year 9 pupils over the course of a few days. In the end 116 out of a cohort of 132 were tested.

The test seemed to be very successful. Only one question provided any concern, and that was on the question of the size of an angle where the same angle was drawn in similar figures. One option allowed for the response: “It is impossible to say”. Some pupils may have judged that it was indeed impossible to measure the angles in the screen. However the question has been left in and analysed accordingly. The only other question that caused a degree of wariness was one inspired by the pattern interference type questions as identified in the literature by Devlin (2000 p40, 63) and Vinner (1997). In some respects this is a “trick” question and it was noticeable that many pupils were not amused by their mistake. However, it has been left in for the analysis.

The basic results for the Year 9 test are attached in Appendix 3.

**3.3 Ascertaining Confidence Levels**

**3.3.1 Ascertaining Confidence Levels Using the YAS package.**

At the same time as assessing the level of misconceptions of a pupil it was also decided to try and determine if pupils were actually aware of their own level of knowledge in answering the questions. To this end an additional piece of code was included that forced the pupil to select a **confidence level** if the particular question was answered. For the Year 7 pupils, and based upon the practice of the LAPT system (<http://www.ucl.ac.uk/lapt/>) a three level scale was used. After the pilot it was felt that pupils were perhaps being a little conservative in choosing the middle option and for the Year 9 pupils a four level scale was used. This was in order to force the pupils to make a decision as to whether they felt confident or not, rather than allowing them to be neutral. Since Confidence Based marking was not being used this seemed reasonable. [As it happens, the feelings about the 3 level scale were shown to be incorrect when the Y7 results were fully analysed and such a scale would have been quite acceptable].

Before the tests began pupils were given an explanation of the purpose of the confidence scale and were asked to be as open and as honest as they could. It was explained that the use of this was for pure research and would not be used in grading them in any way. Pupils seemed unconcerned about this unusual practice. The only issue was in remembering to select an option, but the software would not allow the selection of the next question until one was chosen anyway. The screen pupils saw is shown in figure 3.3.1.1 below.

It is possible to use this level to calculate a confidence based score, but this was not used in the present study.

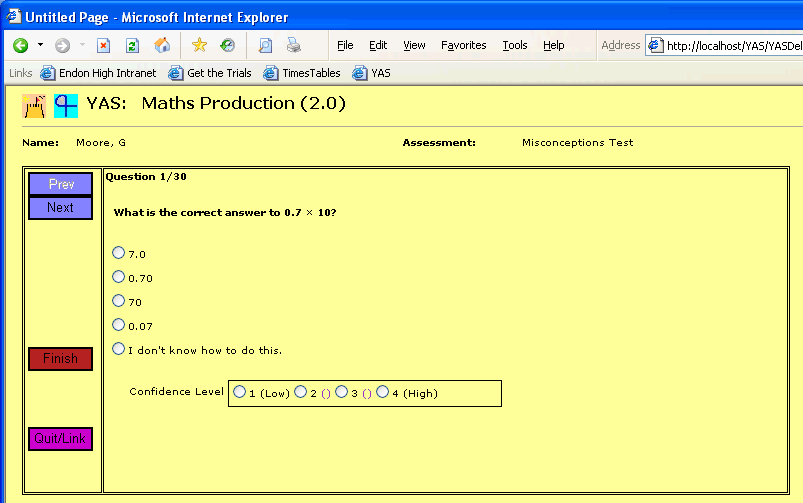


Figure 3.3.1.1 Use of the Confidence Level in a question.

**3.3.2 Ascertaining Confidence by Interview**

An initial analysis of the level of confidence indicated an intriguing situation in that some pupils seemed to express a high confidence in their selected answer even though their selected response was incorrect.

In order to investigate this aspect further it was decided to run a small scale investigation into how pupils were actually thinking and what their thoughts were when they actually answered a few of the questions from the test.

It was decided to audio record some Year 9 pupils from the author’s own form as they did the test. Permission was obtained from the Headteacher to carry this out and letters were sent to all carers of pupils in the form. All parties were informed of the purpose of the study, that pseudonyms would be used and that the audio recordings would be destroyed at the end of the research (Bell 1999 p45-6). They were told that a fellow pupil would be in view as the recording was done.

There was a surprising amount of negativeness to the research by the pupils. It had been felt that they would be quite intrigued and find it novel. In the event fifteen consent forms were obtained, one refusal and seven non-respondents. The pupils in the form are placed there in Year 7 by the head of year. The criteria does not include any analysis of academic ability, accordingly they have a wide range of academic abilities from level 4 to level 8.

In order to conduct the interviews it was decided to use “Think Aloud” protocols as described in Kupermintz, Le and Snow (1999). In this process the interviewees are asked to think aloud whilst completing the test items. The interviewer tries hard not to intervene except to remind the pupils to think aloud during moments of silence. The prompt “Can you tell me what you are thinking of, or feeling now?” is used.

Initially it was also hoped to gain some insight into emotional signalling as predicted in Skemp’s Theory of Intelligence, and so a prompt sheet was produced with some emotion words on it to help pupils. In the event this was found to be more of a hindrance than a help, because pupils were focussed on the screen rather than on the prompt sheet and after the first few interviews the sheet was dispensed with.

The actual process involved the pupil sitting at the desk with a laptop in front of them and the logon screen for YAS displayed. The pupils wore a headset with a microphone, although the headset didn’t cover the ears. The laptop was set up to record the pupils as they spoke. A pocket digital recorder was also used concurrently for backup purposes. As it happened this was useful for discerning responses that were not clear from the headset microphone.

The pupils would log on and start the test. Although the intention had been to do the whole test, it was immediately apparent that this was impractical so the test was reduced to just the first ten questions: on place value, integers, fraction and decimal equivalence, vertical subtraction, perimeter and median.

Although the goal had been to be very strict with simply asking the prompt at moments of silence, it was obvious that more flexibility had to be used. Some pupils are naturally garrulous whereas other seem barely able to communicate. It was also found useful at times to prompt the pupil a little more, as it was hoped that they might reveal more of their thinking and if aided to see if they could actually obtain a correct response to a test item. At times the interview became more free flowing than “think aloud” would allow for.

The transcripts are attached in appendix 5.

**3.4 Effect of Different Feedback Strategies on Remediating Misconceptions**

For the final part of the research it was determined to analyse the effect of different feedback strategies on remediating the misconceptions.

YAS can provide a variety of feedback responses from none at all, to an indication that a response is correct or incorrect, to a sentence explaining the error and right up to a web link that allows a web page to be displayed showing full details of the error and possibly even linked into some kind of tutoring system.

YAS can also allow pupils, after having submitted a test, to go through the questions and see how well they have answered a question. If the No Feedback flag is not set they can see which questions they get right and if so configured why an answer might be incorrect. However all pupils, no matter what the flags, do get to know their overall score.

As described in the literature review, concepts and schema are held to be resistant to change and stable and that therefore it is unlikely that simple telling will generally result in remediation. However this needs to be proven and so it was decided to compare the effects of three different, simple feedback strategies and see if there is any significant impact in the treatments.

The three types of feedback are:

Group A. No Feedback. For an individual question no feedback is provided on the response to any question. They do not know what questions they got right or wrong.

Group B. Response Correct/Incorrect. The pupil can go through each question and see if their response was correct or incorrect for the question. So they know which questions they answered incorrectly, though not why.

Group C. Feedback on Response. The pupil can get an explanation of why their response is incorrect. This takes the form of a sentence or two displayed at the bottom of the screen.

In order to carry out the experiment it was decided to look at another cohort of pupils who had reached Year 9 (i.e. the previous year 8 pupils from when the first part of the research was carried out) and do a test with the specific treatment for a pupil and then a week or so later a post test using the same test in order to evaluate the significance of the treatment.

All year 8 pupils at Endon take the National Optional Year 8 tests, which are essentially the SATs for Year 8. Pupil do sit different tiers of entry for this test. The entire cohort of 147 pupils was ranked on the basis of this test, by mark and tier, and each group of three pupils from the top down, was randomly grouped into either Group A, B or C. By this it was hoped to create three groups of generally similar ability range. The pupils’ MidYIS maths scores were used to check that the groups were comparable to each other (at a 95% confidence level) and confirmed this hypothesis.

The YAS system was set up to deliver the appropriate type of feedback for each pupil. This test had the confidence level option turned off. Over the course of a few days every pupil was given the test. By the end of the period quite a few pupils had been absent and these were rounded up by some ICT Associate teachers and given the test, so that in all, almost the entire cohort was given the test. A small amendment was made to one question before the trial, in that the option for “It is impossible to tell” for the angles question was removed. Although this is a potentially damaging thing to do when comparing the two year 9 cohorts, it was felt that the test would be better with this change. Also the formulation for one of the options for the question on the circumference was changed from π x 2r to the more usual 2 x π x r.

The test took place the week before a one week half term holiday and it had been planned and anticipated that all pupils would again take the test in the week they came back. In the event the test was not administered to pupils until the second week back. So pupils had a gap of between two and three weeks between the tests. All pupils were again administered the test, but this time all three groups were given the full feedback option. This was to counter any ethical objections as to some pupils not getting feedback on the pre-test (Peate and Franklin 2002, Buchanan 2000).

Again almost every single pupil was given the test and only two pupils in the entire cohort missed both.

The basic results for this test are included in appendix 6.

**Chapter 4. Evaluation.**

**4.1 General Level of Misconceptions Found in Year 9.**

The (first) Year 9 cohort was given the 30 question misconceptions test as described. With relation to the test instrument itself it was felt that this was a particularly pleasing test as it seemed accessible to the lower ability pupils and also challenged the brightest pupils.

The mean result of the test was 56% with a standard deviation of 20%.

More complex analysis can be performed on the test and the questions, such as measuring the facility of the question. This is generally the percentage of pupils in a test who get a question right. A good question should have a facility of about 60% (TAL), though a test should contain questions with a range of facilities to allow less able pupils a chance to get some correct and allow more able pupils to be identified (Clausen-May 2001). However floor and ceiling effects must be guarded against whereby all pupils get the question right or none do (Hodson, Saunders and Stubbs 2002).

Figure 4.1.1 shows a stem and leaf plot showing the facilities of the questions.

0 38 key 3|4 means 34% of pupils answered a question

1 05 correctly

2

3 24

4 18

5 12667789

6 11566

7 278

8 033367

9

Figure 4.1.1 Stem and Leaf showing facility of questions.

It can be seen that most questions had a reasonably good facility, though the questions on the formula for the area of a circle and expanding the quadratic were clearly very challenging.

More complex analyses can be performed on questions, such as finding the discrimination (CAA Centre) and calculating the point biserial correlation coefficients (Clausen-May 2001), but these are not done here.

The year 9 classes are set based upon their performance in the optional Year 8 tests and upon the teachers’ judgements. Figure 4.1.2 is a table showing the overall results of the misconceptions test by class

|  |  |  |  |
| --- | --- | --- | --- |
| **Class (Year 9)** | **Mean** | **SD** | **Number in class** |
|  |  |  |  |
| Set 1 (top) | 78 | 9 | 31 |
| Set 2 | 63 | 10 | 28 |
| Set 3 | 49 | 15 | 27 |
| Set 4 | 38 | 9 | 20 |
| Set 5 (bottom) | 28 | 9 | 10 |
|  |  |  |  |
| **Total** | **56** | **20** | **116** |

Figure 4.1.2 Results by class

This indicates that the misconceptions test is discriminating the ability of the pupils quite well and corresponds with the overall sets.

The results of the 30 question basic misconceptions test was correlated with the performance of the pupils on the Year 9 Mock SATs papers (from the previous year 2004). Because the SATs papers are offered to pupils at different levels and because the levels are only a gross measure of ability it was decided to “decimalise” the SATs results enabling all pupils to be considered as one group. Although there is some overlap of questions, it cannot be guaranteed that a level 5.6 from the 4-6 paper corresponds exactly with a 5.6 from the 5-7 paper for instance. However, it is felt that any variation is small.

The percentage result from the misconceptions test was compared against the mock SATs result as seen below in figure 4.1.3. This is for 114 pupils (No mock SATs level was available for two pupils.



Figure 4.1.3 Y9 Misconceptions versus Mock SATs grade

The degree of positive correlation between the two measures is quite remarkable. The correlation coefficient is **0.91** (to 2 d.p.) using Excel’s built in PEARSON function.

Although it was expected that in general the more misconceptions a person has the worse their test results would be, it was not expected that such a simple test would reveal that the level of simple misconceptions would seem to have such a substantial effect upon performance in a complex test like the end of key stage 3 test. This is especially so since the test instrument does not cover many misconceptions commonly noted and not in the same proportions as questions in the four basic areas of number, algebra, shape and space and handling data found in the end of KS3 test.

The results were later correlated with the actual SATs for 2005 and the correlation coefficient was found to be **0.89.**

As an experiment the linear trend line equation of

**SATs level = 0.545\*Misconception% + 3.4252**

was used to predict the Mock SATs level of the next Year 9 (2005-6) cohort. The results are as below in figure 4.1.4:



Figure 4.1.4 Projected Grade versus Actual Grade for Mock SATs (2nd Y9 Cohort)

As can be seen the degree of correlation is quite reasonable (0.80 Pearson). This seems to indicate that a simple and cheap test can reasonably predict results in an expensive and complex test. The average difference was +0.2 of a level, but with a standard deviation of 0.6 (a little larger than hoped for).

There are quite a few outliers, especially at the low end. This would seem to indicate that the formula as obtained from the first trial is not discriminating enough for lower ability pupils. Note also, that there is nothing to suggest a linear relationship exists. Perhaps a logarithmic relationship might be more appropriate, though there is no theory to indicate this as yet. This needs further research.

Although perhaps not as accurate as was hoped, the test instrument does show potential and with perhaps more questions, more data to work with and a better model for prediction, it would prove to be a useful tool in the teacher’s target setting arsenal.

Next, the responses for the pupils from the second cohort of Year 9 pupils – the ones involved in the remediation experiment were compared against the results from the first Year 9 cohort. The results for the original Y9 test, the pre and post tests for the second cohort and some questions from the pilot Y7 test are compared in the table in appendix 4.

What is quite remarkable, comparing say the original year 9 and the second Year 9 pre test, is that for many questions the same proportions of pupils were choosing the same option.

For instance on question 3 : What is 1 – 0.07, 22% of pupils in both cohorts felt it was 0.03, 13% and 16% felt that it was 1.03 and 57% in both chose the correct response. Similarly with question 13: What is 2 divided by 8, 61 and 59% chose the correct answer and 29% in both cohorts chose 4 as a being the correct response.

Although the proportions don’t always show such similarity, they are generally close enough as to be indicators that something very interesting is going on. Why should such similar proportions of pupils exhibit similar selections?

Even more intriguingly is that those questions for which a Year 7 test equivalent exists, a similar proportion is noted there as well. Further data analysis is required for this. Perhaps using an “intelligence” parameter such as the MidYIS score may reveal some linkage.

From the first Y9 test it would seem that the performance of pupils is determined to a major degree by the basic misconceptions they have in mathematics. Although some errors in the SATs test may be due to slips or processing errors, this analysis is revealing quite clearly that it is not just limited problem solving ability that is holding up pupils’ progress and achievement.

From a teacher’s perspective this indicates that much more effort is required in identifying these misconceptions in each pupil and remediating them thoroughly. A practical method could be to simply identify to the pupils the sorts of misconceptions that arise as the concept is being taught. One of the problems is that since the misconceptions identified by this test are quite basic (or rather fundamental) then remediating them can slow down pace through the curriculum and of course the progress of those pupils who do not have the misconception may then be held up. This is why the use of CBA in remediating misconceptions would be so advantageous.

The use of this multi-choice misconceptions test with the YAS program (or similar) is highly valuable in identifying who exactly has the misconception and what form it takes. The next section looks at this issue.

**4.2 Analysis of Confidence Levels**

One of the major features of YAS as a computer based assessment system is the ability to carry out immediate analysis of a test and to analyse down to a pupil’s individual responses.

For instance the Question Analyser program can break down the responses to an individual question and display the results graphically. By clicking on a particular bar the system can identify which actual pupils have answered a particular option. A screen dump is shown below in figure 4.2.1

To the right of each bar can be seen a number. This indicates the confidence that pupils had in answering a particular option. There were four levels available. The option selected was converted to a percentage, so that 0% indicates no confidence, 33% not very confident, 66% quite confident and 100% very confident.

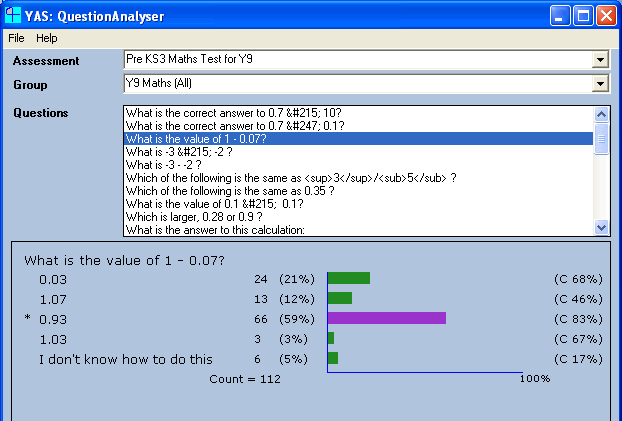


Figure 4.2.1 Screen dump of the Question Analyser showing responses to an individual question.

Appendix 3 shows the full results obtained by this type of analysis. In addition to showing the % Confidence, the figure in brackets after this is the % standard deviation of this confidence. A low value would indicate little variation in confidence and a higher value indicates that confidence was more dispersed. In addition to that a further analysis on the data was carried out resulting in the table below (Figure 4.2.2).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Q.No** | **% Correct** | **Av Conf** | **SD Conf** |  | **0** | **1** | **2** | **3** |  | **% Wrong** | **Av Conf** | **SD Conf** |  | **0** | **1** | **2** | **3** |
| **1** | 83 | 94 | 14 |  | 0 | 1 | 16 | 83 |  | 17 | 77 | 27 |  | 0 | 20 | 30 | 50 |
| **2** | 35 | 72 | 31 |  | 5 | 20 | 30 | 45 |  | 65 | 60 | 29 |  | 7 | 29 | 42 | 22 |
| **3** | 61 | 83 | 28 |  | 3 | 12 | 18 | 67 |  | 39 | 60 | 30 |  | 9 | 26 | 42 | 23 |
| **4** | 79 | 87 | 19 |  | 0 | 3 | 33 | 64 |  | 21 | 78 | 29 |  | 4 | 13 | 29 | 54 |
| **5** | 41 | 80 | 22 |  | 0 | 9 | 41 | 50 |  | 59 | 73 | 26 |  | 1 | 18 | 40 | 40 |
| **6** | 60 | 82 | 30 |  | 6 | 9 | 18 | 67 |  | 40 | 52 | 33 |  | 16 | 33 | 31 | 20 |
| **7** | 66 | 84 | 23 |  | 3 | 3 | 36 | 59 |  | 34 | 61 | 26 |  | 5 | 26 | 51 | 18 |
| **8** | 56 | 69 | 25 |  | 0 | 25 | 43 | 32 |  | 44 | 67 | 31 |  | 4 | 29 | 29 | 37 |
| **9** | 82 | 92 | 19 |  | 1 | 3 | 14 | 82 |  | 18 | 60 | 29 |  | 5 | 33 | 38 | 24 |
| **10** | 83 | 96 | 14 |  | 1 | 0 | 8 | 91 |  | 17 | 93 | 17 |  | 0 | 5 | 10 | 85 |
| **11** | 48 | 97 | 12 |  | 0 | 2 | 5 | 93 |  | 52 | 98 | 7 |  | 0 | 0 | 5 | 95 |
| **12** | 32 | 90 | 17 |  | 0 | 3 | 24 | 73 |  | 68 | 82 | 24 |  | 3 | 6 | 34 | 57 |
| **13** | 62 | 78 | 26 |  | 3 | 10 | 38 | 49 |  | 38 | 81 | 31 |  | 5 | 16 | 9 | 70 |
| **14** | 58 | 90 | 21 |  | 2 | 5 | 15 | 79 |  | 42 | 60 | 28 |  | 4 | 34 | 40 | 21 |
| **15** | 51 | 89 | 23 |  | 2 | 7 | 15 | 76 |  | 49 | 70 | 27 |  | 4 | 18 | 45 | 34 |
| **16** | 15 | 67 | 35 |  | 12 | 18 | 29 | 41 |  | 85 | 86 | 24 |  | 2 | 6 | 22 | 69 |
| **17** | 63 | 76 | 27 |  | 3 | 14 | 35 | 48 |  | 37 | 59 | 35 |  | 17 | 15 | 41 | 27 |
| **18** | 68 | 93 | 14 |  | 0 | 1 | 17 | 82 |  | 32 | 73 | 27 |  | 3 | 16 | 41 | 41 |
| **19** | 3 | 89 | 19 |  | 0 | 0 | 33 | 67 |  | 97 | 69 | 29 |  | 6 | 19 | 39 | 36 |
| **20** | 72 | 96 | 14 |  | 0 | 2 | 8 | 89 |  | 28 | 89 | 20 |  | 0 | 6 | 22 | 72 |
| **21** | 67 | 93 | 16 |  | 0 | 4 | 12 | 84 |  | 33 | 69 | 36 |  | 13 | 13 | 26 | 47 |
| **22** | 77 | 77 | 25 |  | 1 | 13 | 38 | 47 |  | 23 | 83 | 21 |  | 0 | 7 | 37 | 56 |
| **23** | 87 | 91 | 19 |  | 1 | 3 | 17 | 79 |  | 13 | 76 | 27 |  | 7 | 0 | 53 | 40 |
| **24** | 11 | 86 | 22 |  | 0 | 8 | 25 | 67 |  | 89 | 84 | 25 |  | 4 | 4 | 28 | 64 |
| **25** | 56 | 82 | 24 |  | 2 | 8 | 33 | 58 |  | 44 | 69 | 33 |  | 8 | 20 | 30 | 42 |
| **26** | 84 | 87 | 21 |  | 1 | 5 | 25 | 69 |  | 16 | 63 | 32 |  | 11 | 17 | 44 | 28 |
| **27** | 63 | 80 | 23 |  | 0 | 12 | 35 | 53 |  | 37 | 64 | 32 |  | 10 | 20 | 38 | 33 |
| **28** | 87 | 94 | 17 |  | 1 | 3 | 8 | 88 |  | 13 | 69 | 27 |  | 7 | 7 | 60 | 27 |
| **29** | 54 | 85 | 22 |  | 0 | 8 | 28 | 63 |  | 46 | 58 | 34 |  | 14 | 27 | 31 | 27 |
| **30** | 8 | 81 | 24 |  | 0 | 11 | 33 | 56 |  | 92 | 93 | 18 |  | 2 | 1 | 14 | 83 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **Avg** | **57** | **85** | **21** |  | **2** | **7** | **24** | **67** |  | **43** | **72** | **27** |  | **6** | **16** | **33** | **45** |

Figure 4.2.2 Analysis of Confidence Levels

This shows for each question the percentage that selected the correct answer and the percentage that selected the wrong answer. It ignores any pupils who said they did not know how to answer a particular question as no confidence is allowed for that option.

For each set of answers the table shows the average confidence selected, where 0% = no confidence at all and 100% is full confidence. It also shows the standard deviation of these confidences. The table then shows the percentage breakdown for each of the four options selectable, where 0 is the lowest confidence (i.e. 0%) and 3 is the highest confidence (i.e. 100%). The means for each column are then calculated.

What is remarkable is that very low confidence was much less often selected than the higher levels of confidence. In fact the average confidence of 85% for the correct answers and 72% for incorrect answers suggests that generally pupils were selecting options 3 and 4 and this is generally born out by the individual option breakdown.

What was also surprising was the limited spread of confidence. Generally there was a negative skew. Only a few questions revealed any great spread of confidence. It was noticeable that the spread of confidence did seem slightly more pronounced for incorrect answers than for correct ones and as shown above pupils were slightly less confident about their answer than for those who got the correct answer.

What this says is that pupils were generally confident of their answer, even if it was a misconception. The theory had suggested that emotional signals would be generated that would cause disquiet or unease in the pupil. This does not seem to be the case.

In particular, there are some questions that stand out in this regard. For instance Q.10 was the calculation 31-17 displayed as a column subtraction. Although only 17% got this wrong they were clearly very confident (93%) that they were in fact correct. Question 12 which asked which number is between 2.5 and 2.6 showed that the 68% who got it wrong were 82% confident that they were correct. Question 30, a question on precedence of operations (BiDMAS) showed that 92% were locked into a left to right mentality and yet were very confident about their answer. Question 3 on 1 – 0.07 showed that those who answered incorrectly were varied in their confidence. Also question 6, an equivalence to 3/5 showed a disparity in confidence levels.

A profile of each pupils’ confidence rating for questions was produced. This was done by working out their overall average confidence for each question answered. This was also divided into a profile of questions correct and a profile for questions answered incorrectly. (Figures 4.2.3, 4.2.4)



Figure 4.2.3 Confidence profile for Correct Answers (Correlation = 0.46)



Figure 4.2.4 Confidence profile for Incorrect Answers (Correlation = -0.35)

Although it can be seen that the tendency to be confident increases for the more correct answers a person gets, the correlation coefficient (Pearson) is not convincing (0.46). The scatter does also indicate that even those who score poorly are also pretty confident.

The scatter graph for confidence against answers incorrect is even less revealing about any relationship (-0.35 for Pearson). One might have expected that those who got more answers wrong would be less confident, but this does not seem to be revealed here.

For completeness Figure 4.2.5 shows the confidence profile for overall score.

Here there is a more noticeable trend that the higher the score the more confident a pupil is. The Pearson correlation coefficient here is 0.6, which is quite reasonable, but it can also be clearly seen that some pupils are very confident even if their score is poor. There are only a few values at the low score, low confidence end.



Figure 4.2.5 Confidence against Score (Correlation = 0.6)

It can also be said that there are no pupils who lacked confidence and yet obtained a high score.

The actual data for each pupil is attached in Appendix 8. This data has been sorted into Total Score order.

**4.3 Individual Question Analysis**

In the literature review it has been observed how the causes of misconceptions are complex and are often closely related to the actual question type itself.

For example consider place value. A commonly seen misconception is that of 0.7 x 10 = 0.70 as in question 1. A complete analysis of the errors that could be made in this question is multifaceted. However it seems reasonable to assume that one of the main causes is because pupils become aware of the fact that multiplying the whole numbers they commonly meet in primary school by ten results in an additional zero at the end of the whole number. This is valid under the prohabitat (Skemp 1979b) of working with whole numbers. This concept was probably developed by many examples and practice. However it is unlikely that a teacher will have gone into the mechanics of place value exposition with young children. Accordingly they have no underlying ideas on which to relate this observation. The concept is simply: “whenever multiplying by ten add a zero”. 15% of Year 7 pupils felt this was true, 6% in the first cohort of Year 9 pupils and 10% in the second Y9 cohort. Actually this seemed to be a surprisingly small amount considering how often it seems to arise. It would be interesting here to see how these results would change if pupils had to enter the answer themselves.

Another simple example concerns question 3: What is 1 – 0.07 such as might be found in calculating from the law of total probability. The common misconception is to see 0.03 written. This occurred 22% of the time for both Y9 cohorts. It would seem that the number bond concept of 7 + 3 = 10 is being misused here. It is as though the pupils recasts the sum as 10 - 7 = 3 and then shifts the result back? Quite a surprising number seem to ignore the negative sign and just add to get 1.07 [13% and 16%]. It is hard to see where or how the misconception is arising here.

Another common example is found at question 4: where -3 x -2 = -6 is often written. It is quite astounding how many pupils make this error [16%, 24% (34% of year 7)]. It is a perfect example of how examples and practice can sometimes fail to embed the concept. It is difficult to explain the reasons why this should be so, but the refrain: “a negative times a negative is a positive” is one that must resound around all mathematics classrooms. The tape recordings revealed that many pupils seem to have this knowledge. So does the pupil simply see the negatives and pseudo-conceptually just assume that if a negative is involved then a negative must be in the answer? Yet the knowledge that this is not always so must be available surely by Y9.

Questions on fractions also reveal deep seated misunderstandings. For example asking what is the decimal equivalent to 3/5 (Q6). Many pupils think that 0.35 is the decimal equivalent [28%, 31% (36% of Y7)]. This reveals woeful understanding of fractions and decimal place value. These pupils seemingly think that the decimal is simply an alternative way of writing a fraction. How could this have arisen?

Some other findings from looking at individual questions are:

Question 2 involved a division by a 0.1. Only 35% of pupils recognised that dividing by a number between 0 and 1 results in a larger number. However there were signs of lack of confidence in the answer for both those who got it right and those who got it wrong.

Question 5 asked what is -3 - -2. Clearly the subtracting of a negative caused problems for many. 59% of pupils got this wrong. Again there was a wide range of confidence displayed.

Question 12 as already discussed caused many problems. It was striking how many regarded 2.51/2 as a real number and perfectly acceptable, although pupils will have never(?) come across such a presentation.

Question 16 again revealed the left to right linear scanning mode of many pupils. Many took 2x2 to be 2 times x and then squaring. Only 15% of pupils got this right, but displayed a range of confidences.

Question 19, is a higher level question, but it is known for a fact that at least 34 of the pupils have been exposed to squaring brackets and also have had this misconception clearly pointed out to them. (This seems to show that publicly exposing misconceptions is not always a panacea for remediation!)

Question 24 on the correct formula for the circumference of a circle again reveals that pupils have great difficulty with this. In retrospect the way the correct option is posed is unusual and in the remediation test this was changed to the more usual formulation of 2 x π x r rather than π x 2r. However from that test it was noted that only 3% of pupils got this correct as opposed to 11% of pupils in the previous year cohort.

Question 30 was noticeable for the lack of knowledge of precedence of operations.

Clearly much could be said about the other questions as well and a thorough review has revealed disturbing themes that are surprising. In particular the clear lack of familiarity and understanding of fractions and proportion is noticeable and this has already had a major impetus in the author’s own teaching practice.

**4.4 Analysis of Pupil Transcripts**

In order to try and investigate more deeply into what was happening cognitively as the pupils dealt with each question and to try and find an answer to the issue of pupil confidence with the wrong option some pupils were audio recorded as they carried out the first ten questions. It is apparent that there are some common themes among the responses.

**4.4.1 Recognition Factor**

What immediately stands out is that a recognition factor clearly comes into play. This is indicated by a diverse range of expressions such as straightforward responses to the question “do you recognise these sorts of questions?”, to some intriguing responses such as “’cause we haven’t done it in a while” or “we spent a lot of time on the box” (Pupil F), “We did songs with Miss Hill” (Pupil J), “We made up a song about it [circle formula]” (Pupil M), “ I remember a minus times a minus is a positive” (Pupil N), “[Median] – you put them in order and pick the middle one” (Pupil A), and even a sort of negative recognition such as “Oooh I don’t know how you do these” (Pupil B).

This recognition would indicate that concepts are being activated as well as the associated memories (spreading activation) including visual memories: “a grid” (Pupil J) or auditory memories: “a song”. Many pupils though were quite uncertain about how accurately they remembered the aid. It also seemed to lead to an instrumental, surface approach to learning.

One can sense pupils struggling to remember any kind of information that can help them, such as rules: “’cause a minus and a minus equals a plus” (Pupil A), “Because I know that perimeter is adding all the sides up” (Pupil L), “π times d” (Pupil T). Weaker pupils seem relieved to have retrieved some piece of information, but then seem unwilling to consider if there are any other items of information that could help. So for example many pupils recalled the formula for the circumference of a circle as being πr2 (sic), but they didn’t then seem to consider πd or 2πr. It is therefore possible that pupils are often just focussing on the strongest memory trace they have and disregarding or rejecting other weaker memory traces.

Pupils knew they had done certain question types in the past, for instance: “’Cause you know you’ve done it before, but it was a long time ago and you’ve forgot how to do it.” (Pupil G on division by 0.1). This would seem to reveal an instrumental approach whereby the questions have been compartmentalised into separate topics. There did not seem to be any sense of trying to think about the question by trying to reason out how dividing by ten works as in a relational approach. At times pupils even felt that they hadn’t really had as much teaching as they should have had. Pupil F said “It wasn’t gone over dead thoroughly” about circumference, or “we didn’t spend a lot of time on this” about the median.

**4.4.2 Level of Difficulty Identified.**

It was also noted that pupils often associate a level of difficulty with each question. Many found the first question: “What is the correct answer to 0.7 x 10?” to be quite straightforward, “It’s easy” said one. However as soon as they were presented with the second question: “ What is the correct answer to 0.7 ÷ 0.1?” many admitted to uncertainty (e.g. pupil C), and to it being more difficult (e.g. Pupil E). Pupil H said it was “scary!”.

It is interesting to try and determine the considerations and factors that are enabling them to make this judgement. For instance, division is seen as hard, circles are harder than rectangles, subtraction is seen as “easy” (though of course subtraction is quite challenging cognitively).

It was very worrying to observe the number of pupils who felt that the fraction-decimal equivalences were very hard (i.e. “Which of the following is the same as 3/5?”, “Which of the following is the same as 0.35?”). Pupil B said: “Oh god, I don’t like these!”. Nearly half the sample felt that 0.35 is equivalent to 3/5. A few admitted that they didn’t really know (pupil J) or were just guessing (pupil F). The weaker pupils certainly struggled with this. The more able pupils seem to have the key fact that 1/5 equalled 0.2 which allowed them to make progress, whereas the weaker pupils simply had no real comprehension of how fractions and decimals relate.

**4.4.3 Anxiety Level**

The perceived difficulty of a question also seemed to raise the anxiety level quite considerably. Pupil J signalled this quite often, e.g. “All minuses, verrr…”, or it was signalled by nervous laughing or shaking of the head. Long pauses were observed. Pupils would sometimes just move the mouse from one option to the other, sometimes just switching between two options until finally deciding on a particular option (possibly by using probability considerations?). When this behaviour is contrasted with questions they found easy it is seen that then they just got on with it, even if the question involved mental effort, such as the subtraction question and the median question. Pupil E seemed to have quite a lot of test anxiety related to the importance of the test itself and to time issues.

Often, after having made some choice, pupils felt a degree of relief. Pupil K made the comment that they felt “doubt” at first, then after answering the question they felt “calm” or “relief”. In one case they got the answer right, but on another question (circumference) they had exactly the same feeling, but for a wrong selection. It is possible that some pupils do feel anxiety, doubt and unease at first, then after making an internal decision on an answer they feel calm and therefore express confidence. This was typified by pupil G on the -3 - -2 question. The pupil felt the question was quite easy, then incorrectly chose -5 and still felt confident when gently challenged.

**4.4.4 Emotional Attachment**

Some pupils also seemed to put an emotional relationship onto the questions. “Oh I like these” , “Oh I don’t like median.” (pupil B).

It is possible that pupils will express more confidence with questions they like. This is an issue that needs further research.

**4.4.5 Confusion identified**

Many pupils recognised their own level of confusion for some questions. For instance pupil F said: “It’s quite confusing, like all the different ones like range and every thing.” (on median). Pupil J on the perimeter of a rectangle question said: “I get mixed up with perimeters and area…”, and chose the wrong one. Pupil F felt confusion with all the negatives for the “-3 - -2” question and also between the difference between median and mean. Though he clearly knew the algorithms, he couldn’t (at first) correctly associate the words and the procedures. Similarly pupil B couldn’t decide whether the median was the middle or “what’s the most”. When told which one it was she was able to easily complete the task.

**4.4.6 Reasoning Processes and Explanation**

It was interesting to see the reasoning process at work. Pupil J on the median question first chose the seven. When asked to explain her thinking she showed she knew that you had to put them in order and then she realised her mistake. You could see her mentally put them in order and then suddenly the realisation hits. This shows the power of getting pupils to explain their thinking. It seems to force them to think more deeply and carefully about the situation and do things more thoroughly, rather than to simply get it out of the way.

Pupil N, an extremely able mathematician, although at times admitting that some questions were harder, was able to correctly reason his way through a question. For example on question 4 (What is -3 - -2?) the pupil was able to relate mentally to a number line and moving in a direction. This was a good example of true cognitive thinking at work. Pupil E similarly made use of the mental model of a number line effectively (Fishbein 1977). Very few pupils seemed able to work like this, most attempting to use the rule for two negatives (9/14 pupils got this question wrong).

The question on choosing the correct formula for the circumference of a circle often revealed that knowledge could not be adapted. Many pupils knew that the area of a circle was π times r squared. However the formula for the circumference was re-written in a non standard way as π times 2r. It seemed clear that some pupils knew about π times d and two times π times r, but could not adapt to the non-standard presentation. Thereupon they resorted to using the familiar formula for the area, bypassing the fact that this was inappropriate. It was almost as though they were thinking: “Maybe I’ve got it wrong and this (πr2) really *is* the formula for the circumference” - a self delusion. This shows how powerfully a standard presentation can work on people as reported in the literature review.

**4.4.7 Recognition of “silly” mistakes.**

It was also clear that pupils knew they had made a silly mistake when they were shown the answers. Pupil I made only one error on the -3 x -2 question and put -6 though he clearly knew and recognised his error. His observation was that because it was easy for him he hadn’t perhaps looked as closely at the signs as he should have – carelessness. This was similar with pupil H who forgot to order the numbers for the median question and realised his slip when provided with feedback: “Yeah, I know. I forgot about that bit”.

On the “-3 x -2” question pupil J made her first attempt at -6, then suddenly changed, because she remembered the rule of minus times minus makes a plus, she says: “Then, I realised!”. Pupil C also showed this on the question about the equivalent value to 0.35. Firstly she selected 35/10 and was expressing her confidence and then suddenly realised her error. What is interesting about this is that some pupils make an (immediate) response, then some feedback mechanism kicks in and they seem to think and reason *after* the decision. This factor has become more to the fore in the author’s own teaching practice where it has become more noticeable that pupils are often prone to making an immediate answer when asked a question, but invariably it seems to be simply a guess, since when they notice a negative kind of response to their answer they immediately try something else. Mathematics seems to have become a guessing game for them.

**4.4.8 Confidence**

With relation to the question of confidence, more observations were forthcoming about how pupils can be seemingly confident about what they have done and yet choose the wrong answer. Pupil L having struggled with the fractions questions seemed relieved to have to do a subtraction. She simply took a smaller digit from a larger digit, selected the wrong answer and declared she was happy: “Er, this one’s fine, I like stuff like this”. Pupil C said she knew how to do the circumference question and simply chose the wrong formula. She seemed surprised when given feedback on this. Pupils used words like “fine”, “certain”, “straightforward”, “calm”, “satisfied” to describe their positive confidence level.

However when pupils knew they didn’t know the answer they were usually able to describe their level of confidence accurately. Pupil M said that she was “even less confident”, being clearly able to discriminate levels of low confidence. Pupils said they were uneasy”, “unsure”, “not certain”, “stressed”, “I’ve got pressure on me”, “doubt”, “dubious”, At times though, pupils did select the correct answer even if they weren’t confident e.g. pupil J on -3 - -2, pupil H on the circle question. This however seemed rare.

**4.4.9 Mental “Spoonerism”.**

A couple of pupils seemed to get words and concepts mixed up in a particularly strange way. Often this occurred with the swapping of the terms division and negative (Pupil E) or between thinking of “timesing” and yet adding. A sort of cognitive spoonerism? This has become more noticeable in the classroom especially when pupils talk about subtraction and division. They might often say 3 divided by 12, when they really mean 12 divided by 3. It may possibly be linked to the strong left to right association that many pupils seem to have.

**4.4.10 Summary**

Clearly the complexity of pupil responses to answering questions is astounding. Yet these common themes do indicate where misconceptions may be occurring and why confidence is being expressed even when the questions are answered incorrectly.

From a practitioner’s perspective the value of getting children to explain their thinking is paramount. The process often, (but not always), seems to force the pupils to evaluate their responses. It seems to make them slow down and re-consider any immediate first choice they have, or to identify where the weakness is in their understanding. Computer Based Assessment seems to have a weakness in this area as it seems to predispose the pupil to either make a quick decision or to simply make a decision and then quickly move on. The “game playing” mentality of CBA often supports this approach. Somehow CBA needs to develop in a way that enables the more thoughtful, reflective approach to take place. Perhaps adding a delay after a selection is made may encourage the pupil to do this.

**4.5 Analysis of Simple Feedback Strategies in Remediation of Misconceptions**

The final part of the research looked at how it might be possible to remediate the misconceptions using three simple feedback strategies of: no feedback, indicating which question was correct or incorrect and feedback that gave a brief explanation of why the wrong response was incorrect.

In order to determine if a treatment had a significant effect a 1-way ANOVA was performed upon the results. The difference between the pre and post tests was calculated and the Excel plug in Analyse-it was used to perform the ANOVA. The results are shown if figure 4.5.1 below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | analysed with: Analyse-it + General 1.71 | |
| **Test** | **1-way between subjects ANOVA** | | |  |  |  | |
|  | Misconceptions | |  |  |  |  | |
| **Comparison** | Group: A, B, C |  |  |  |  |  | |
| **Performed by** | G Moore |  |  | **Date** | 24 November 2005 | | |
|  |  |  |  |  |  | |  |
| **n** | 137 |  |  |  |  | |  |
|  |  |  |  |  |  | |  |
| **Group** | **n** | **Mean** | **SD** | **SE** |  | |  |
| **A** | 43 | 2.58 | 2.91 | 0.443 |  | |  |
| **B** | 47 | 1.57 | 3.14 | 0.458 |  | |  |
| **C** | 47 | 2.32 | 3.18 | 0.464 |  | |  |
|  |  |  |  |  |  | |  |
| **Source of variation** | **SSq** | **DF** | **MSq** | **F** | **p** | |  |
| **Group** | 24.91 | 2 | 12.46 | 1.31 | 0.2732 | |  |
| **Within cells** | 1274.17 | 134 | 9.51 |  |  | |  |
| **Total** | 1299.08 | 136 |  |  |  | |  |

Figure 4.5.1 ANOVA on Feedback Strategies

For an ANOVA the data should be continuous and although the differences are whole numbers the test “could” have been marked to a real value. Since there are more than 30 in each group the data can be considered to be normally distributed and the results are independent of each other. The actual pre and post test scores are attached in Appendix 6.

The null hypothesis is that no treatment has any significant (at the 95% level) impact upon the post test results.

The probability of the F value being 1.31 is 0.27 and therefore it is unlikely that there is any significant variation between the groups.

Looking at the data results closely there are a few anomalies that are difficult to explain. Below is a table (figure 4.5.2) that shows the number of pupils for each change in score from pre-test to post-test.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Mark Change | **-6** | **-5** | **-4** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| Number | 1 | 2 | 3 | 4 | 4 | 9 | 12 | 24 | 19 | 16 | 16 | 13 | 1 | 4 | 4 | 2 | 0 | 1 | 1 |

Figure 4.5.2 Change in marks

As can be seen there were some exceptional changes in scores. A couple of pupils increased their score by over 10 marks and a few had their scores decrease by 5 marks or over. Looking at the individual pupils the one who gained 11 marks is in the bottom set and was in Group B. It seems hard to explain how he did so well. The one who gained the most was in group C. Even more interestingly is that of the pupils in the 8 and 9 mark increase 4 out of the 6 are in the author’s own class. This class has worked on fractions and negative numbers during the timescale of this investigation, but not between the pre and post tests. However, the worst performing pupil is also in this class.

The table below (figure 4.5.3) shows the change in marks broken down by treatment group. It is hard to see any significant differences between the three groups (as the ANOVA indicates). As before though the range of marks is quite remarkable.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Mark Change A | **-6** | **-5** | **-4** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| Number | 1 | 0 | 0 | 1 | 0 | 3 | 2 | 8 | 5 | 7 | 7 | 3 | 1 | 1 | 1 | 2 | 0 | 0 | 0 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Mark Change B | **-6** | **-5** | **-4** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| Number | 0 | 2 | 1 | 3 | 2 | 2 | 4 | 10 | 7 | 7 | 1 | 5 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Mark Change C | **-6** | **-5** | **-4** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| Number | 0 | 0 | 2 | 0 | 2 | 4 | 6 | 6 | 7 | 2 | 8 | 5 | 0 | 2 | 2 | 0 | 0 | 0 | 1 |

Figure 4.5.3 Mark changes broken down by treatment group.

In conclusion it can be seen that these simple types of feedback seem to have very little impact upon remediation. This is in accord with the theory and from the evidence seen in this research. Misconceptions are resistant to change and a more cognitively demanding approach needs to be taken. How CBA can do this is therefore still open to question.

**Chapter 5. Conclusions**

This research has focussed on the means by which misconceptions arise and how they might be detected and remediated. The overriding concern for this is to attempt to improve pupils’ performance and achievement.

This research has discovered that the level of misconceptions a pupil has, as determined by using even a simple and narrowly focussed test instrument, seems to have a significant impact in a pupil’s progress and achievement in high stakes tests.

Using CBA to administer and analyse the level of misconceptions in a pupil would seem to be a vital tool in a teacher’s teaching arsenal. It is suggested that more work is done in developing these tests and assessing their effectiveness in uncovering misconceptions. It is felt to be very important that a secondary school should use a similar test instrument for all new year 7 pupils and then implement targeted remedial action at identified pupils.

Class teachers should use similar tests, perhaps on a topic by topic basis to identify particular misconceptions at the beginning of a teaching episode so that they can be sure to target specific areas as required.

Class teachers should be therefore made more aware of how misconceptions might arise in teaching and be on the lookout for explaining these issues to classes. For instance the author now takes care to present mensuration questions using a variety of orientations to stop pupils thinking in terms of standard orientations of diagrams.

It has been noted quite clearly that the way misconceptions arise is complex and varied. It has not been possible to simply categorise misconceptions by type. Each misconception needs to be fully analysed and considered. It would be useful if the literature on individual misconceptions could be meta-analysed so as to provide question writers with an easily accessible source of information and allow the use of specific and plausible distractors in multi-choice questions.

The theories of Skemp have been used to provide a basis for discussion, but in one important aspect have been found to be seemingly deficient in that the emotional signals that are expected to be generated do not seem to be so easily identified by school pupils themselves. Indeed this research has shown that the feeling of confidence in a given answer may be misplaced. This would appear to have implications for Confidence Based Marking schemes, for younger students at least.

The ability of CBA to provide a simple remediation tool by use of simple feedback strategies has been found to be deficient. It would appear that, as the literature suggests, a more cognitively intrusive and demanding position must be taken with remediation. It is not at all clear what form that should take with CBA and whether it can be done simply and cheaply.

It has been suggested by anecdotal evidence from a colleague, that a form of buddy testing may prove useful. In this two pupils work together to solve problems (from physics in the colleague’s case). It has been observed that pupils discuss and argue about the correct response before selecting an option. Of course there are difficulties in assessment here, but the idea looks promising.

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**Appendix 1 Some common misconceptions from the literature and practice.**

The following brief notes are taken from the papers indicated and some from my own experience. Quite a few of the ideas influenced my selection for the test items.

Misconceptions (Personal) Things that make me cry.

x2 = 2x

3.6 × 10 = 3.60

a(b+c) = ab + c

3x – 2 = 4, so 3x = 4-2, 0r 3x = 2 so x = 3/2

-3 × -2 = -6

3 – 4 = 1

32

-17

----

25

3.14 said “three point fourteen”

2.62/3 2.6.34



2x – x = 1x

Probability of 6 = 1-6

0 × 2 = 2

3 × 18 written as 3

×18

---

Not cancelling down 4/8

What is the name of this shape :

Any gon but the right gon



Spelling: sircel, sfear isosles parallgram equilatrel fourty

Using diamond for rhombus

7% of £26 = 26/7 x 100

4/7 = 0.47 or 4.7

0.3 = 1/3

0.56 = 1/56

1¾ = 13/4

42/7 = 42/7

3/8 + 2/3 = 5/11

36000 = 3.6 × 103 because 3 zeros



23% of x = x/23



Probability 2-3, 2:3 2 out of 3 anything but 2/3

P(rain) = 4/7, P(not rain) = 7/4

P(rain) = 0.02, P(not rain) = 0.08

Write thirty two million, four hundred and seventy thousand and twenty two in words: 32,470,22

32 + 34 = 36 or 38

Counting on includes first number e.g. 9-7 so 7,8,9 = 3

48

× 26

---

8288

4

7% = 0.7

£2.06 = 2.6 on a calculator

* 1. × 0.1 = 0.2



What number is between 2.5 and 2.6 Ans 2.5½

Time 3 lots of 1hr 35min = 3 x 1.35 = 4.05 = 4hr 5min

40 ÷ 2 said as two divided by forty

Which is larger 0.28 or 0.8, ans 0.28



**Olivier A.** : 1989, Handling Pupils’ Misconceptions, Presidential Address delivered at Thirteenth National Convention on Mathematics, Physical Science and Biology Education, Pretoria, 3-7 July 1989, web: <http://academic.sun.ac.za/mathed/MALATI/Misconceptions.htm>

1) What order of difficulty do we expect in the following four additions for young pupils learning column addition?

(A) 523 (B) 593 (C) 586 (D) 586

+25 +25 +25 +25

For (A) some responses were

(E) 523 (F) 523 (G) 523

+25 +25 +25

1. 948 48

2) e + f = 8, what is e + f +g? (58% of std 6 (Y7) pupils produced a numerical answer)

Most common responses were: 12 (4+4+4), 15 (3+5+7), 15 ( 8+g and g is the 7th letter)

3) Which number is largest? (A) 0.62, (B) 0.234, C(0.4) (D) 0.532

Response 0.62 (38%), 0.532 (29%), 0.4 (25%)

4) x2 – 10x + 21 = 12

so (x-7)(x-3) = 12

so x-7=12 or x-3=12

so x=19 or 15.

5) 263 546

-128 -375

1. 231

6) √(a+b) = √a + √b

(a+b)2 = a2 + b2

a(bc) = (ab)(ac)

7) What calculations required for:

(A) 1 litre of petrol costs £1.12. How much will it cost to fill a tank holding 3 litres?

(B) 1 litre of petrol costs £1.12. How much will it cost to fill a tank holding 0.53 litres.

Success rate of 27% for (B) (63% chose to divide by 0.53) .

From 1 kg of wheat you get 0.75 kg of flour. How much flour from 15 kg of wheat?

1. 1kg of detergent is used to make 15kg of soap. How much soap can be made from 0.75kg of detergent?
2. obtained 79%, (B) obtained 27% (45% choosing division)

8) In a certain college there are six times as many students as there are professors. Use S for the number of students and P for the number of professors to write an equation for the situation.

P = 6S (Using the S and P as labels as in 6g is 6 grams)

9) Replacement: 4 x 4 becomes 4+4

23 becomes 2 x 3

6 ÷ ½ becomes 6 ÷ 2 or 6 × ½

3x/x becomes 2x

New skills can also interfere with earlier skills:

x + x = 2x. Then x × x = x2 learnt, so now x + x becomes x2!!

# Fischbein and Muzicant in Tall and Thomas (eds) 2002 p.53-77

Differences in solving the equation x/2 + x/3 = 10, or simplifying x/2 + x/3

(So many will do 3x + 2x = 10 or 60 for the equation and 3x + 2x instead of (3x +2x)/6 for the expression.

# Tall D. in Tall and Thomas (eds) 2002 p 156-157

5 - -2 = 7. How can taking away make bigger?

Fraction times fraction, how can multiplying make smaller?

3 + 2x + 1 = 6x, the “numbers” have an addition between them

xn means n lots of x multiplied together, so what does x½ mean?

# Thomas M.O.J. . in Tall and Thomas (eds) 2002 p 187

47% of a sample of 13 year olds thought 6÷7 and 6/7 were different, 6÷7 is a sum, 6/7 a fraction.

# Stacey K., MacGregor M. . in Tall and Thomas (eds) 2002 p226-7

Conflict of Hindu-Arabic notation : if a = 2 and b = 7 then a x b can be written as 14, ab, ba, 2b, a7, 7a, but not as 27 or 72. [if x = 3 7x = 73, not as 7x3=21] p226

Association of letters with their alphabetic positions, e.g. give a height 10cm greater than h cm, ans: 18 or r. because h is 10th in the alphabet or r is 18th (8+10) P227

A blue card costs 5 cents and a red card costs 6 cents. I buy some red cards and some blue cards. The total cost is 90 cents.

Let b stand for the number of blue cards and r stand for the number of red cards, which of the following is correct. Tick your choice or choices.

1. b = 5, r = 6.
2. 5b+6r = 90
3. b+r = 90
4. b+r = 11
5. 6b+10r =90
6. 12b + 5r = 90
7. b+r = $0.90
8. none of these.

Only 7% of 188 Year 10 (Australia) pupils chose the correct answer of (b). p229

# White & Mitchelmore in Tall and Thomas (eds) 2002 p241-3

m5 ÷ m3 = m2 , so 65 ÷ 63  = 12

30-60% of secondary students judge angle size on the basis of such features as the length and orientation of the arms and radius of the arc marking the angle. P243

**Cooper B., Dunne M**. : 2000, Assessing Children’s Mathematical Knowledge. Social Class, Sex and Problem Solving, Open University Press, ISBN 0 335 20316 7

Realistic problem issues:

An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed? Children commonly gave non-whole number answers…seen as inappropriate by test designers for counting buses. P 25. But what if the answer was for purposes of splitting costs, or what if another camp was also using buses and we wanted to know what fraction was required for the soldiers? P 29

(S problem) Steve has bought 5 planks of 2m each. How many planks of 1m can he saw out of these planks?

(P problem) Steve has bought 4 planks of 2.5m each. How many planks of 1m can he saw out of these planks? P26 (From Verschaffel et al 1994)

Realistic considerations e.g. loss of material due to sawing, versus paper and pencil problem in a school maths problem context. Child should understand in the S problem that approximately 1m will do and yet draw on realism to answer P problem. P depends on what planks are for. If shelves then need complete pieces, but if for flooring does it matter if two half m pieces are used?

Grandfather gives his four grandchildren a box containing 18 balloons, which they share equally. How many balloons does each get?

4.5 or 4? (p.30)

**Hall R.D.G**.: 2002, An Analysis of Thought Processes During Simplification of an Algebraic Expression, Philosophy of Mathematics Education 15, <http://www.ex.ac.uk/~PErnest/pome15/processes.htm>

[an analysis of students responses to simplifying an algebraic expression:

Simplify:



180 Y9,10,11 in selective Bermuda school. 41 correctly cancelled and stopped, 20 factorised, but did not cancel factor of (n-2), 44 simply cancelled the x^2, 5 went on to cancel the x’s to get 5/4]

If they felt they could cancel the x’s, why not the x2 at the beginning?

The expression seemed to act as a trigger for the process of factorise then cancel,

One pupil said “I don’t like negative answers, so I preferred my method which got a positive answer”

3 ¼ is 3 + ¼ so is 3x the same as 3 + x (Matz 1982)

**Swedosh P**. : 1999, Dealing With Diverse Student Backgrounds, The Challenge of Diversity Delta 99, Symposium on Undergraduate Mathematics, Laguna Quays, Brisbane. <http://www.sci.usq.edu.au/staff/spunde/delta99/deltaon2.htm>

Simplify expressions 1-3 as fully as possible. (common misconception on right)

1) 100!/98! 2!

2) 3x x 3x 92x, 3x^2

3) 2x + 2x 4x

Solve for x in equations 4-7

4) x2 = 81 x=9

5) x2 – 4x = 0 x=4

6) x2 = x x=1

7) 1/x – 1/b = 1/a x = a+b

8) Solve for x: 2x+4 < 5x+10 x < -2; x>2

9) Factorise (2x+y)2 – x2 3x2 +4xy + y2

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4/5 + 4/5 = (8/10)

½ of 20 = ¼ of ? (10)

1/3 of 60 = 2/3 of ? (120)

**Devlin** (2000 p40, 68) The Math Gene, Weidenfeld and Nicholson, ISBN 0-465-01618-9 provides these examples:

A farmer has twelve cows. All but five die. How many remain?

Tony has 5 balls, which is 3 fewer than Sally. How many balls does Sally have?

In connection with younger children aged about three or four, a child when shown 2 red apples and 3 red bananas and asked how many different kinds of fruit there are will answer “5”. The sense of number is stronger than the abstractness of the question. By five this is properly answered.

Certainly the presentation of information can lead to difficulties. For instance it was noted that when asking student teachers to identify the intersection of the altitudes of a triangle they would invariably put it inside the triangle even if the triangle was obtuse.

There are of course many other examples and it would be useful to begin collecting these and publishing them. Perhaps even providing reasons as to why they are occurring.

**Appendix 2 Questions and Results from Year 7 Pilot Study.**

**No % Confidence**

1. What is the perimeter of this rectangle? (7 by 5)

 12 6 8% (58%)

 35 23 29% (76%)

 24 50 \* 62% (91%)

 70 1 1% (50%)

 I don't know how to do this. 0 0% (0%)

2) What is the correct name for this shape? (a regular pentagon)

 Triangle 0 0% (0%)

 Quadrilateral 0 0% (0%)

 Hexagon 9 11% (61%)

 Pentagon 71 \* 89% (87%)

 I don't know

3) What is the value of -3 times -2 ?

 +6 38 \* 48% (67%)

 -6 27 34% (65%)

 -5 7 9% (57%)

 +5 3 4% (50%)

 -1 1 1% (50%)

 +1 2 2% (25%)

 I don't know how to do this. 2 2% (25%)

4) What is the correct value of (4/7)

 0.47 29 36% (40%)

 0.571... 10 \* 12% (55%)

 4.7 34 42% (38%)

 1.75 4 5% (25%)

 I don't know 4 5% (0%)

5) What is the correct way to say 3.14

 Three point fourteen 11 14% (82%)

 Three point one four 66 \* 82% (85%)

 Three hundred and fourteen 1 1% (0%)

 Any of the above 0 0% (0%)

 I don't know 2 2% (50%)

6) What is the correct answer to 3.6 x 10

 3.6 0 0% (0%)

 3.60 12 15% (79%)

 36 61 \* 75% (91%)

 360 6 7% (50%)

 I don't know how to do this 2 2% (50%)

7) In algebra, what is the correct answer to 2x - x

 1x 23 29% (37%)

 x 12 15% (17%)

 -x 11 14% (23%)

 2x - x, the same. 24 30% (33%)

 I don't know how to do algebra. 10 12% (40%)

**No % Confidence**

8) What is the correct answer to this fraction sum: 3/4 + 2/3

 17/12 7 \* 9% (21%)

 5/12 21 26% (43%)

 5/7 48 60% (57%)

 32/43 3 4% (17%)

 I don't know how to do fractions 1 1% (0%)

9) What is the answer to this calculation: 31  
 - 17

-------

 26 1 1% (100%)

 14 71 \* 88% (94%)

 16 2 2% (75%)

 24 7 9% (79%)

 I can't do this 0 0% (0%)

10) I want to calculate 23% of £70, which calculation would I do.

 70 divided by 23 then times 100 18 22% (39%)

 70 divided by 23 26 32% (29%)

 23 divided by 70 11 14% (36%)

 70 divided by 100 then times 23 19 \* 24% (53%)

 I can't do percentages 6 8% (33%)

11) Which expression below is the same as 3(x + 4)

 3x + 4 58 72% (47%)

 3x4 7 9% (21%)

 3x + 12 2 \* 2% (0%)

 7x 6 7% (33%)

 I can't do algebra 8 10% (25%)

12) What fraction below is the same as 0.37?

 3/7 2 3% (50%)

 37/100  53 \* 67% (77%)

 37/10 20 25% (45%)

 7/3 1 1% (0%)

 I can't do this 3 4% (17%)

13) What is the standard form for 36000?

 36 x 103 45 56% (49%)

 3.6 x 103 10 12% (35%)

 3.6 x 105 10 12% (30%)

 3.6 x 104 2 \* 2% (25%)

 I don't understand Standard Form 13 16% (27%)

14) What is the best answer below to 32 + 35

 32 + 35 25 \* 32% (42%)

 32+5 8 10% (25%)

 37 25 32% (46%)

 310 8 10% (19%)

 I don't understand this 13 16% (27%)

**No % Confidence**

15) Which decimal is the same as 7%

 0.7 32 40% (61%)

 7.0 12 15% (58%)

 0.07 34 \* 42% (75%)

 0.700 1 1% (0%)

 I can't do this 2 2% (50%)

16) A calculator display shows 2.6 after a money calculation. What does it mean?

 2 pounds and 6 pence 22 28% (70%)

 2 pounds and 06 pence 7 9% (71%)

 26 pounds with 0 pence 0 0% (0%)

 2 pounds and 60 pence 48 \* 61 (90%)

 I don't understand this 2 3% (50%)

17) Which is larger: 0.28 or 0.6?

 0.28 12 15% (42%)

 0.6 69 \* 85% (84%)

 They are the same size 0 0% (0%)

 I don't know 0 0% (0%)

18) What number is half way between 2.7 and 2.8?

 2.75 74 \* 91% (91%)

 2.71 2 2% (75%)

 2.78 1 1% (0%)

 2.705 4 5% (75%)

 I don't know 0 0% (0%)

19) I change 267 minutes into hours and minutes using a calculator. The display shows 4.45. What time does this represent?

 4 hours and 45 minutes 76 95% (76%)

 445 hours 0 0% (0%)

 Just less than 4 and a half hours 1 \* 1% (100%)

 A mistake has been made 3 4% (50%)

 I don't know 0 0% (0%)

Q20 had an error in it.

**Appendix 3 Questions and results from the Year 9 Misconceptions and Confidence Study. (April 2005) No % Confidence**

1) What is the correct answer to 0.7 × 10?

 7.0 96 \* 83% (94%)(14)

 0.70 7 6% (62%)(30)

 70 5 4% (93%)(15)

 0.07 8 7% (79%)(25)

 I don't know how to do this. 0 0%

2) What is the correct answer to 0.7 ÷ 0.1?

 0.07 66 57% (62%)(28)

 0.70 7 6% (43%)(32)

 7.0 40 \* 34% (72%)(31)

 70 0 0%

 I don't know how to do this 3 3%

3) What is the value of 1 - 0.07?

 0.03 25 22% (68%)(30)

 1.07 15 13% (44%)(21)

 0.93 66 \* 57% (83%)(28)

 1.03 3 3% (67%)(58)

 I don't know how to do this 6 5%

4) What is -3 × -2 ?

 -6 19 16% (81%)(28)

 -5 3 3% (67%)(33)

 +6 91 \* 78% (87%)(19)

 +5 2 2% (67%)(47)

 I don't know how to do this 1 1%

5) What is -3 - -2 ?

 -1 46 \* 41% (80%)(22)

 -5 29 26% (75%)(29)

 +5 18 16% (70%)(23)

 +1 20 18% (73%)(26)

 I don't know how to do this 0 0%

6) Which of the following is the same as 3/5?

 0.35 32 28% (55%)(32)

 0.3 7 6% (43%)(32)

 0.53 6 5% (44%)(40)

 0.6 67 \* 58% (82%)(20)

 I don't know 4 3%

7) Which of the following is the same as 0.35 ?

 35/10 19 17% (63%)(22)

 3/5 20 17% (58%)(30)

 5/3 0 0%

 35/100 75 \* 65% (84%)(23)

 I don't know 1 1%

**No % Confidence**

8) What is the value of 0.1 × 0.1?

 0.1 27 23% (69%)(36)

 0.01 65 \* 56% (69%)(25)

 10 2 2% (33%)(0)

 0.2 22 19% (67%)(23)

 I don't know how to do this 0 0%

9) Which is larger, 0.28 or 0.9 ?

 0.28 15 13% (67%)(31)

 0.9 93 \* 80% (92%)(19)

 They are both the same size. 3 3% (44%)(19)

 It depends on the situation. 3 3% (44%)(19)

 I don't know 2 2%

10) What is the answer to this calculation: 31

-17

----

 26 8 7% (96%)(12)

 14 96 \* 83% (96%)(14)

 16 3 3% (100%)(0)

 24 9 8% (89%)(24)

 I can't do this 0 0%

11) A farmer has twelve cows. All but five die. How many cows does the farmer now have?

 12 0 0%

 7 60 52% (98%)(7)

 5 56 \* 48% (97%)(12)

 0 0 0%

 I don't know 0 0%

12) Which of the following numbers is between 2.5 and 2.6 ?

 2.51/2 76 66% (82%)(24)

 2.7 0 0%

 2.505 37 \* 32% (90%)(17)

 2.65 1 1% (100%)(0)

 There is no number between these two 1 1% (33%)(0)

13) What is the answer to 2 divided by 8 ?

 1/4 71 \* 61% (78%)(26)

 4 34 29% (89%)(24)

 2.8 4 3% (50%)(43)

 You can't do this sort of sum 5 4% (53%)(38)

 I don't know 2 2%

14) What is the answer to 3/5 + 1/10 ?

 7/10 66 \* 57% (90%)(21)

 4/15 37 32% (63%)(27)

 4/5 9 8% (44%)(29)

 3/50 1 1% (67%)(0)

 I don't know how to do this. 3 3% (22%)(38)

**No % Confidence**

15) Which of the following is the same as 7% ?

 0.7 37 32% (68%)(29)

 7 12 10% (78%)(22)

 1/7 7 6% (62%)(23)

 0.07 59 \* 51% (89%)(23)

 I don't know 0 0%

16) If x = 3 what is the value of 2x2 ?

 12 12 16% (65%)(35)

 36 73 63% (92%)(16)

 529 3 3% (67%)(0)

 18 17 \* 15% (67%)(35)

 I can't do algebra. 5 4%

17) What is the answer to p2 × p3 ?

 p5 71 \* 61% (76%)(27)

 p6 22 19% (61%)(34)

 p23 3 3% (44%)(51)

 2p5 16 14% (60%)(35)

 I can't do this. 4 3%

18) Simplify the following expression 3(x + 4)

 3x + 4 30 26% (73%)(24)

 3x + 34 4 3% (58%)(50)

 3x + 12 77 \* 66% (93%)(14)

 It depends on what value x is. 3 3% (89%)(19)

 I cannot do algebra. 2 2%

19) Which of the options is the same as (a + b)2

 a2 + 2ab + b2 3 \* 3% (89%)(19)

 a2 + b2 83 72% (73%)(27)

 2a + 2b 8 7% (62%)(38)

 2ab2  9 8% (44%)(29)

 None of these, it can't be done 8 7% (54%)(31)

 I can't do algebra 4 3%

20) What is the **perimeter** of this rectangle? (7 by 5)

 35 cm 27 23% (90%)(18)

 75 cm 0 0%

 12 cm 5 4% (80%)(30)

 24 cm 83 \* 72% (96%)(14)

 I don't know 1 1%

21) What is the area of this triangle? (h=5, b=6 right)

 11 cm2 11 9% (67%)(33)

 30 cm2 21 18% (83%)(27)

 22 cm2 6 5% (28%)(39)

 15 cm2 77 \* 66% (93%)(16)

 I don't know how to do this. 1 1%

**No % Confidence**

22) Which of these two angles is larger? (same size, but A drawn smaller)

 A is larger 0 0%

 B is larger 7 6% (81%)(26)

 They are both the same size 89 \* 77% (77%)(25)

 It is impossible to know 20 17% (83%)(20)

 I don't know 0 0%

23) What is the correct name for this shape? (Pentagon)

 Quadrilateral 2 2% (83%)(24)

 Pentagon 103 \* 86% (91%)(19)

 Hexagon 13 11% (74%)(28)

 Octagon 0 0%

 I don't know 1 1%

24) What is the correct formula for the circumference of a circle?

 π × r2 77 66% (89%)(18)

 π × r 14 12% (64%)(40)

 π × 2r2 8 7% (71%)(33)

 π × 2r 12 \* 10% (86%)22)

 I don't know 5 4%

25) What is the median of the set of numbers : 3, 7, 9, 2, 2?

 2 17 15% (70%)(35)

 3 64 \* 56% (82%)(24)

 7 12 11% (67%)(28)

 9 19 17% (68%)(34)

 I don't know 2 2%

26) A fair coin is tossed 9 times. It comes up heads every time. Which of the following statements is true.

 The coin must come up tails next because it is a fair coin 0 0%

 This coin is not a fair coin, otherwise tails would have

come up more times. 13 11% (62%)(30)

 On the next throw a tail is more likely than a head. 5 4% (67%)(41)

 There is an evens chance of heads coming up again. 96 \* 83% (87%)(21)

 I don't know 1 1%

27) I catch 3 fish, my two friends catch 2 fish each. What is the mean number of fish we catch?

 5 divided by 3, i.e. 1.66666... 12 10% (69%)(33)

 7 divided by 3, i.e. 2.33333... 68 \* 59% (80%)(23)

 5 divided by 2 i.e. 2.5 23 20% (68%)(31)

 7 divided by 2, i.e. 3.5 5 4% (33%)(24)

 I don't know 7 6%

28) A taxi can take 4 people. How many taxis are needed to take 18 people to the theatre?

 3 taxis 0 0%

 4 taxis 10 9% (70%)(33)

 5 taxis 105 \* 87% (94%)(17)

 9 taxis 5 4% (67%)(0)

 I don't know 0 0%

**No % Confidence**

29) You are told that a number x is > -2. Which of the following is true.

 x could be -1 60 \* 52% (85%)(22)

 x could be -2 20 17% (35%)(30)

 x could be -3 26 23% (74%)(29)

 x could be any number you want 5 4% (60%)(28)

 I don't know 4 3%

30) What is the answer to 3 + 4 × 2?

 9 0 0%

 11 9 \* 8% (81%)(24)

 14 105 91% (93%)(18)

 16 1 1% (67%)(0)

 I don't know 1 1%

**Note**: figures in brackets refer to the % confidence and the %standard deviation of the confidence.**Appendix 4 Comparison of results for two Year 9 Cohorts**

**Misconceptions Results Study with Y9 04/05 05/06 (y7 05)**

**Pre post**

1) What is the correct answer to 0.7 × 10?

 7.0 83%\* 84% 85% (75%)

 0.70 6% 10% 8% (15%)

 70 4% 2% 3% (7%)

 0.07 7% 1% 4%

 I don't know how to do this.

2) What is the correct answer to 0.7 ÷ 0.1?

 0.07 57% 61% 54%

 0.70 6% 18% 14%

 7.0 34%\* 18% 30%

 70 1% 2%

 I don't know how to do this 3% 1% 1%

3) What is the value of 1 - 0.07?

 0.03 22% 22% 15%

 1.07 13% 16% 14%

 0.93 57%\* 57% 65%

 1.03 3% 1% 4%

 I don't know how to do this 5% 3% 1%

4) What is -3 × -2 ?

 -6 16% 24% 25% (34%)

 -5 3% 3% 3% (9%)

 +6 78%\* 69% 68% (48%)

 +5 2% 3% 4% (4%)

 I don't know how to do this 1%

5) What is -3 - -2 ?

 -1 41%\* 46% 41%

 -5 26% 37% 41%

 +5 16% 1% 8%

 +1 18% 16% 9%

 I don't know how to do this 1% 1%

6) Which of the following is the same as 3/5?

 0.35 28% 31% 23%

 0.3 6% 4% 3%

 0.53 5% 3% 6%

 0.6 58%\* 62% 68%

 I don't know 3% 1%

7) Which of the following is the same as 0.35 ?

 35/10 17% 12% 12%

 3/5 17% 16% 10%

 5/3 1%

 35/100 65%\* 71% 77%

 I don't know 1% 1%

8) What is the value of 0.1 × 0.1?

 0.1 23% 39% 33%

 0.01 56%\* 39% 49%

 10 2% 3% 2%

 0.2 19% 19% 16%

 I don't know how to do this

9) Which is larger, 0.28 or 0.9 ?

 0.28 13% 10% 7% (15%)

 0.9 80%\* 83% 90% (85%)

 They are both the same size. 3% 2%

 It depends on the situation. 3% 4% 3%

 I don't know 2% 1%

10) What is the answer to this calculation: 31

-17

----

 26 7% 8% 5% (1%)

 14 83%\* 82% 85% (88%)

 16 3% 5% 6% (2%)

 24 8% 5% 1% (9%)

 I can't do this

11) A farmer has twelve cows. All but five die. How many cows does the farmer now have?

 12 2% 1%

 7 52% 46% 32%

 5 48%\* 52% 66%

 0

 I don't know

12) Which of the following numbers is between 2.5 and 2.6 ?

 2.51/2 66% 54% 47%

 2.7 2% 2%

 2.505 32%\* 37% 49%

 2.65 1% 1% 1%

 There is no number between these two 1% 5% 1%

13) What is the answer to 2 divided by 8 ?

 1/4 61%\* 59% 65%

 4 29% 29% 28%

 2.8 3% 6% 4%

 You can't do this sort of sum 4% 5% 2%

 I don't know 2% 1% 1%

14) What is the answer to 3/5 + 1/10 ?

 7/10 57%\* 62% 68%

 4/15 32% 29% 27%

 4/5 8% 8% 4%

 3/50 1% 1% 1%

 I don't know how to do this. 3%

15) Which of the following is the same as 7% ?

 0.7 32% 24% 24% (40%)

 7 10% 14% 13% (15%)

 1/7 6% 2% 3% (42%)

 0.07 51%\* 60% 59%

 I don't know

16) If x = 3 what is the value of 2x2 ?

 12 16% 18% 16%

 36 63% 59% 57%

 529 3% 5% 4%

 18 15 \* 17% 21%

 I can't do algebra. 4% 1% 2%

17) What is the answer to p2 × p3 ?

 p5 61%\* 49% 55%

 p6 19% 31% 22%

 p23 3% 3% 4%

 2p5 14% 15% 17%

 I can't do this. 3% 1% 2%

18) Simplify the following expression 3(x + 4)

 3x + 4 26% 35% 36% (72%)

 3x + 34 3% 4% 4%

 3x + 12 66%\* 54% 63% (2%)

 It depends on what value x is. 3% 5% 3%

 I cannot do algebra. 2% 1% 1%

19) Which of the options is the same as (a + b)2

 a2 + 2ab + b2 3%\* 10% 13%

 a2 + b2 72% 61% 65%

 2a + 2b 7% 9% 11%

 2ab2  8% 16% 7%

 None of these, it can't be done 7% 3% 2%

 I can't do algebra 3% 1% 1%

20) What is the **perimeter** of this rectangle? (7 by 5)

 35 cm 23% 24% 16% (29%)

 75 cm 2% 1%

 12 cm 4% 8% 7% (8%)

 24 cm 72%\* 66% 75% (62%)

 I don't know 1%

21) What is the area of this triangle? (h=5, b=6 right)

 11 cm2 9% 14% 6%

 30 cm2 18% 41% 38%

 22 cm2 5% 3% 7%

 15 cm2 66%\* 41% 50%

 I don't know how to do this. 1%

22) Which of these two angles is larger? (same size, but A drawn smaller)

 A is larger

 B is larger 6% 4% 7%

 They are both the same size 77%\* 24% 17%

 It is impossible to know 17% 72% 80%

 I don't know removed

23) What is the correct name for this shape? (Pentagon)

 Quadrilateral 2% 3% 3%

 Pentagon 86%\* 82% 85% (89%)

 Hexagon 11% 16% 11% (11%)

 Octagon 1%

 I don't know 1%

24) What is the correct formula for the circumference of a circle?

 π × r2 66% 59% 59%

 π × r 12% 19% 14%

 π × 2r2 7% 7% 14%

 π × 2r 10%\* 7% 11%

 I don't know 4% 7% 3%

25) What is the median of the set of numbers : 3, 7, 9, 2, 2?

 2 15% 16% 9%

 3 56%\* 61% 72%

 7 11% 5% 4%

 9 17% 18% 15%

 I don't know 2% 1%

26) A fair coin is tossed 9 times. It comes up heads every time. Which of the following statements is true.

 The coin must come up tails next because it is a fair coin 2% 4%

 This coin is not a fair coin, otherwise tails would have

come up more times. 11% 5% 11%

 On the next throw a tail is more likely than a head. 4% 3% 6%

 There is an evens chance of heads coming up again. 83%\* 88% 83%

 I don't know 1% 2%

27) I catch 3 fish, my two friends catch 2 fish each. What is the mean number of fish we catch?

 5 divided by 3, i.e. 1.66666... 10% 22% 14%

 7 divided by 3, i.e. 2.33333... 59%\* 50% 64%

 5 divided by 2 i.e. 2.5 20% 17% 12%

 7 divided by 2, i.e. 3.5 4% 7% 9%

 I don't know 6% 3% 2%

28) A taxi can take 4 people. How many taxis are needed to take 18 people to the theatre?

 3 taxis 1% 3%

 4 taxis 9% 10% 6%

 5 taxis 87%\* 87% 91%

 9 taxis 4% 1% 1%

 I don't know 1%

29) You are told that a number x is > -2. Which of the following is true.

 x could be -1 52%\* 57% 71%

 x could be -2 17% 11% 8%

 x could be -3 23% 21% 16%

 x could be any number you want 4% 9% 3%

 I don't know 3% 2% 1%

30) What is the answer to 3 + 4 × 2?

 9 2% 1%

 11 8%\* 22% 30%

 14 91% 74% 67%

 16 1% 1% 2%

 I don't know 1%

**Notes:**

The first column is 116 pupils responses from the Year 9 cohort 04/05 who did the test first with confidence recording.

The second and third columns are 147 pupils from the Year 9 cohort 05/06 who did a pre test and post test (without confidence recording) Note that Q22 had the impossible to say option removed and Q24 had the π x 2r changed to 2πr.

The fourth column contains results from the pilot with Y7. Not all questions line up exactly and so this is just for indications rather than true comparison.**Appendix 5 Pupil Interview Transcripts (July 2005)**

**Pupil A (Female)** Score : 3/10 (KS3 SATs Level : 5)

T) Okay so there’s the first question. (1) What is the correct answer to 0.7 x 10?) What are, do you feel, thinking when you see that?

A) Errrm, fine about it.

T) Okay, choose whichever you want (7.0 √)…click on next, now what about this question, what, question two (2) What is the correct answer to 0.7 ÷ 0.1?), what are you feeling about that?

A) Erm it’s a bit harder than the last one.

T) What makes it harder?

A) Erm, ‘cause its divide.

T) Okay, (0.07 ×) [pause] okay. What about that one (3) What is -3 x -2?), what are you feeling with that one?

A) Erm, its okay.

T) Do you recognise it?

A) Ermm, a little bit from …

T) Do you know anything about negative times negative?

A) Erm equals a plus.

T) Okay, how confident are you with your answer? (+6 √)

A) Erm, fairly confident.

T) Okay, what about this one (4) What is -3 - -2?), what are you feeling now?

A) Erm fairly confident, [pause] now erm.

T) What made you change your mind?

A) Because I knew it was a plus and not a minus, ‘cause a minus and minus equals plus (+1 ×).

T) Okay question five now (5) Which of the following is the same as 3/5?)

A) [Very long pause] Not so confident on this one.

T) Okay, (0.35 ×) [long pause] (6) Which of the following is the same as 0.35?) what are you feeling on this one now?

A) It’s very similar to the last question, sum, but the opposite like.

T) Okay, [pause] done many of these before?

A) Not loads, but quite a few.

T) Okay [very long pause] What are you feeling now?

A) I’m not quite sure at all. (3/5 ×)

T) Not quite sure? It’s taken you a bit of time to… what other one were you thinking of?

A) Erm, that one, the top one.

T) Thirty five over ten. What about this question? (7) What is the answer to 31 – 17 (vertical arrangement)?)

A) Erm it’s, it’s okay, but I’d rather have like pen and paper [“ter”].

T) Okay, unfortunately we haven’t got any paper.

A) [laughs] [sv] (26 ×) [very, very long pause]

T) What are you thinking on this question, question eight? (8) What is the perimeter of this rectangle (7 by 5)?)

A) Erm, [long pause] I, I’ve seen them before, but I aren’t quite sure.

T) What word, what hits you, what are you thinking of?

A) Seven times five (35 ×)

T) [???] (9) Which is the correct formula for the circumference?) [pause] What are you thinking now?

A) Uhm, pi and think about a circle pi r squared.

T) Okay. Is there anything in the words that helps you there? Any key words?

A) Circle.

T) Circle.

A) Circumference.

T) Circumference. You’re feeling fairly confident on that one?

A) Yeah (πr2 ×)

T) Okay, (10) What is the median of the set of numbers 3, 7, 9, 2, 2?) [pause] what are you thinking now?

A) [very long pause+ sv] Not quite sure.

T) Not sure? Okay. Do you know anything about the er median?

A) You put them all in order, then the middle number. (3 √)

T) Okay [pause] so just click on finish.

**Pupil B (Female)** Score : 4/10 (KS3 SATs Level : 4)

T) There’s question 1 (1) What is the correct answer to 0.7 x 10?) Tell me what you’re feeling when you see that question?

B) Er confused

T) Confused? Have you seen questions like that before?

B) Yeah

T) Yeah

B) Ermm

T) So which one do you think it might be, and why do you think that?

B) [Long pause] That one (0.07 ×).

T) How are you feeling now?

B) Er, better

T) Better [laughs]. Now that you’ve done it, what about this one, question two, (2) What is the correct answer to 0.7 ÷ 0.1?) [???] seven divided by 0.1?

B) Errr, errr, that one again (0.07 ×).

T) That one again. So what are you feeling?

B) I don’t know how you work them out.

T) So you just don’t know?

B) Yeah

T) So any of those [words?]?

B) Unsure.

T) Unsure, okay. Negative three times negative two? (3) What is -3 x -2?)

B) [Long pause] errr, plus five, no I don’t know.

T) Okay, alright, so what sort…?

B) It’s either negative five or plus five, ‘cause three times two is five and there’s a minus and a minus, which gives a plus, so its plus five.

T) So you know that rule then, and what’s three times two?

B) five, six

T) ahh!

B) Ohh!

T) What are you feeling now? Yes you can change it.

B) Wronggg!

T) [Laughs]

B) Plus six.

T) So did, did you feel any sort of recognition suddenly?

B) I know how you work it out like (+6 √).

T) (4) What is -3 - -2?) This is negative three take away negative two

B) So [twa?]

T) What are you feeling now?

B) [Long pause] Bit unsure.

T) Bit unsure?

B) It’s [someat] one I think. It’s plus one, [sv “I don’t know, but I’ll try it”] (+1 ×)

T) How are you feeling now you’ve answered that question?

B) [Pause] Same as all the others. Bit relieved like.

T) Okay so here’s fractions questions (5) Which of the following is the same as 3/5?)

B) Oh god! I don’t like these.

T) So you don’t, what are you feeling?

B) I don’t know how you do ‘em, so…

T) Right, so, worried?

B) Erm, [pause] [laughter], er that one, no.

T) [???]

B) Nought point, just try that one cause I don’t know it. (0.35 ×)

T) So that’s just a random guess?

B) Yeah.

T) Okay.

B) Ermm, (6) Which of the following is the same as 0.35?) [long pause] I don’t have a clue what that one is.

T) You don’t have a clue?

B) So I just guessed it. (35/10 ×)

T) What about question seven? (7) What is the answer to 31 – 17 (vertical arrangement)?)

B) Er

T) Do you recognise those sort of questions?

B) Yeah, ermm, […] fourteen, I think. (14 √)

T) Okay, how are you feeling about that one? (8) What is the perimeter of this rectangle (7 by 5)?)

B) Sureish, but…

T) Sureish?

B) Yeah alright [long pause]. Ohh I don’t know how you do theeese

T) Right.

B) I think you add all the numbers up.

T) Okay.

B) fourteen fiftee, twenty four.

T) Okay. What made you think that’s what you had to do, add all the numbers up?

B) ‘Cause, there’s going to be a seven there and a five there, so is like seven add seven, five add five.

T) Good, and what made you decide to do that rather than say multiply them?

B) ‘Cause it’s a perimeter.

T) So what do you think the perimeter means?

B) The numbers at the side, round, like [sv “can aren’t ah”] (24 √)

T) [???] feeling okay about that. So question nine? (9) Which is the correct formula for the circumference?)

B) Ermm

T) Seen these before? Circumference?

B) I don’t have a clue so, so just don’t know, don’t have a clue (I don’t know) [long pause] (10) What is the median of the set of numbers 3, 7, 9, 2, 2?) Oh I don’t like median.

T) Do you remember what it means?

B) Yeah, you have to put all the numbers in a line. It’s either what’s the most of them or it’s the middle number.

T) Right so you can’t think which one it is?

B) Sit down and be two [???] [pause] or seven?

T) Okay

B) No three, or seven, don’t know.

T) If I was to tell you it’s the middle…

B) Oh it’s…

T) How are you feeling now that you know that?

B) Better, two, two, three, seven, nine. It’ll be three (3 √)

T) Okay, so how are you feeling about that?

B) Better, finish?

T) Well done, thank you very much.

**Pupil C (Female)** Score : 7/10 (KS3 SATs Level : 7)

T) So what are you thinking, feeling when you see this question? (1) What is the correct answer to 0.7 x 10?)

C) Erm, that, that I know how to do it (7.0 √)

T) Okay, and you feel quite confident?

C) Yeah.

T) What are you feeling now? (2) What is the correct answer to 0.7 ÷ 0.1?)

C) I think I, er, we’ve done it, just got to think about it, I’m not quite sure [very long pause]. (0.07 ×)

T) How confident were you about that answer?

C) Not very.

T) Not very. Okay, what about this one, number three? (3) What is -3 x -2?)

C) Erm, I think, I’d be able, I can, I’ll be able to do it.

T) Okay. Is there anything you know about it that comes to your mind?

C) Erm that a minus times a minus is a plus. (+6 √)

T) Good, okay [long pause]. What are you feeling with this one? (4) What is -3 - -2?)

C) Erm, [pause], feel kinda confident, I think.

T) Okay [very long pause]. How confident are you about that one?

C) I’m alright, I think. I think it’s right, but I’m not sure. (+5 ×)

T) [Long pause] What are you feeling now with this one? (5) Which of the following is the same as 3/5?)

C) Erm, I know how to do it, I’m quite confident on that one. (0.6 √)

T) You’ll have to speak up a bit now with all the noise.

C) Okay.

T) What about this one, number six? (6) Which of the following is the same as 0.35?)

C) Erm, yeah, I know how to do this one.

T) How confident are you about that?

C) Quite confide, er no its not that…

T) Ah, what made you change then?

C) I got it wrong, ha!

T) How did you know you had got it wrong?

C) ‘Cause it’s not out of ten, it’s out of a hundred. (35/100 √)

T) What are you feeling with this question? (7) What is the answer to 31 – 17 (vertical arrangement)?)

C) Yeah, I know, I know how to do that one. (14 √)

T) Right okay, this one? (8) What is the perimeter of this rectangle (7 by 5)?)

C) Yeah, I know how to do perimeters so I’m quite confident [long pause] (24 √)

T) Okay, well done [pause]. What are you feeling with this one, number nine? (9) Which is the correct formula for the circumference?)

C) Uhm, yeah, I know how to do this one [very long pause] (πr2 ×)

T) What about this last one? (10) What is the median of the set of numbers 3, 7, 9, 2, 2?). What are you thinking now?

C) I’m just thinking, which one, how to do a median, it’s, it’s the middle, middle one isn’t it?

T) Say that again.

C) It’s the middle one.

T) The middle one?

C) So it’s, put them in order, [long pause], be three. (3 √)

T) Okay, so just click on finish and just say yes. That’s not bad, 70%. You were wrong about the circle.

C) Oh, was I?

T) It’s the area.

C) Oh.

T) But most of the rest of them were alright, and you got that division one wrong as well. So how are you feeling about that?

C) Alright

T) Do you feel okay

C) Yeh.

T) Right, thanks very much, appreciate your help on that.

**Pupil D (Female)** Score : 6/10 (KS3 SATs Level : 7)

T) So, what are you feeling when you see that question? (1) What is the correct answer to 0.7 x 10?)

D) Erm, confident. (7.0 √)

T) So select what you think the answer is, okay, and then go to next. What about that one? (2) What is the correct answer to 0.7 ÷ 0.1?). What are you feeling about that one?

D) Erm, bit confused.

T) Okay, what makes you feel a bit confused?

D) Erm, [long pause], because I got to divide des, decimals.

T) Okay.

D) Erm [long pause].

T) So you don’t know how to do that one?

D) No (I don’t know how to do this).

T) How about that one? (3) What is -3 x -2?). How are you feeling on question three?

D) Ermm, bit awkward ‘cause I’m, I find negatives a bit difficult.

T) Have you seen that before, that sort of thing?

D) Yeah

T) Okay. Can you remember anything about it?

D) I think two minus, no…

T) Go on.

D) I think two minuses makes a plus, but I’m not sure, so I’ll just click that. (+6 √)

T) What are you feeling with that? (4) What is -3 - -2?)

D) Erm. Awkward again ‘cause its decimals again.

T) Okay. Is it decimals?

D) [Long pause], no you know what I mean, negatives, so that’s the one. (I don’t know how to do this)

T) So you just don’t know how to do that one?

D) No.

T) How are you feeling with that one? (5) Which of the following is the same as 3/5?)

D) Ermmm, okay. (0.6 √)

T) So you’re quite happy with fractions and decimals?

D) Yeah.

T) What are you feeling with that one? (6) Which of the following is the same as 0.35?)

D) Ermmm, it’s okay. (35/100 √)

T) Okay, what about that one? (7) What is the answer to 31 – 17 (vertical arrangement)?)

D) It’s quite easy. (14 √)

T) Quite easy. What method are you using to try and solve it?

D) Erm, [pause], I just count on from seventeen to thirty one.

T) So what about this question? (8) What is the perimeter of this rectangle (7 by 5)?)

D) It’s quite easy (24 √) [long pause] (9) Which is the correct formula for the circumference?) Ermm, okay, I think. (πr2 ×)

T) What about the last question? (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

D) Erm its, easy, I think. Is that it? (2 ×)

T) So how are you feeling now, that the test is over?

D) Okay, relieved

T) Thank you very much.

**Pupil E (Male)** Score : 10/10 (KS3 SATs Level : 7)

E) Is this an important test at all?

T) No no, not at all, why does that make a difference?

E) Yeah

T) [Laughs]

E) [Laughs]

T) How do you feel now you know it’s not important?

E) Er, relieved.

T) Relieved, ok, right. No this is just for research.

E) Uh uh.

T) So what are you feeling when you see that question? (1) What is the correct answer to 0.7 x 10?)

E) Er, quite confident because I know how to do it. (7.0 √)

T) Okay, so if you select your correct answer and click on next. What about this one? (2) What is the correct answer to 0.7 ÷ 0.1?)

E) Errrm, bit uncertain, not quite used to it, ‘cause it’s both negatives.

T) When you say negatives, what do you mean?

E) Er, well, divide sorry, not negatives.

T) Divide, right.

E) Which find more difficult that timesing.

T) Have you seen these types of questions before then?

E) Occasionally, not very often [sv, “er that one”]. (7.0 √)

T) How confident do you feel about that answer?

E) Er, middlish.

T) Okay, [pause]

E) What, do you want, er feel quite confident on this. (3) What is -3 x -2?)

T) Okay, [pause]. What made you feel confident on it?

E) Because I’ve been taught it and I remember it. (+6 √)

T) Good, what about that question? (4) What is -3 - -2?)

E) Erm, bit uncertain.

T) Why’s that?

E) Well, you see it and you see it’s all negatives, so it like triggers something, but, it’sss, [sv “one”] but that’d be adding. Is there like a time limit at all?

T) No, no, take as much time as you want.

E) Yeh.

T) There’s no rush.

E) Okay.

T) What made you change? (-1 √)

E) Why? because I realised, that, uh, the answer’s gotta be one somewhere, so I think it’s a minus one, because if you’re adding a minus you’re going that way.

T) Okay

E) But if you minus a minus.

T) What, number line?

E) Yeah, [pause] (5) Which of the following is the same as 3/5?) Three fifths, er, I think a fifth is 0.2, so times 0.2 by 3, you get 0.6.

T) So that was some information…

E) Yeah [sv “???”] that one. (0.6 √)

T) How do you feel about that one? (6) Which of the following is the same as 0.35?)

E) Quite confident (35/100 √) [long pause] erm.

T) Okay what were you feeling with that question (7) What is the answer to 31 – 17 (vertical arrangement)?)

E) Erm, easy cause I’ve seen it laid out like that before. (14 √)

T) Right, how did you go about solving it?

E) Er…

T) What’s your method?

E) I added ten then added four, I just went up.

T) I see, right, okay (8) What is the perimeter of this rectangle (7 by 5)?)

E) [sv “7, 14, 28”] Er, easy. (24 √)

T) Okay, why do you think it’s easy?

E) Because, I know how to do perimeters.

T) Right, okay, you don’t get confused with area?

E) No

T) Okay, good. (9) Which is the correct formula for the circumference?)

E) [sv “circle, erm, ha”]

T) What are you thinking on this one?

E) Well, that’s area, pi r squared, so I’m trying to work it out.

T) Okay

E) So it can’t be that one, so it must be one of these.

T) Okay, so the second option or the fourth option.

E) Erm that one. (π x2r √)

T) Okay, what made you choose that one?

E) Okay, I realised that one’s the diameter, two times the radius.

T) Okay.

E) [sv “??? Easy one”] Erm

T) What are you feeling about this one? (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

E) Oops, ermmm, all right, I got to put them in order [pause], one two, sooo, three. (3 √)

T) Okay, so you’re happy with that?

E) Yeah, so click finish.

T) Ten out of ten, well done, first person so far I think.

E) Makes a change

T) How are you feeling now?

E) Alright, but not overly bothered, seeing it wasn’t important, er, like, it wasn’t like SATs or anything like that.

T) What do you think would have happened if it had been important?

E) I’d have been a lot more relieved that I got ten out of ten.

T) [Laughs] How do you think it would have worked with your problem solving, made a difference?

E) Uhm, not too much of a difference as long as done preparation an’ stuff like that.

T) Thanks very much.

**Pupil F (Male)** Score : 5/10 (KS3 SATs Level : 6)

T) What are you thinking with this one? (1) What is the correct answer to 0.7 x 10?)

F) Uhmm, fairly simple.

T) Fairly simple?

F) So so. (7.0 √)

T) What, so you’re feeling what, confident, happy?

F) Uhmm yeah that’s fairly easy.

T) Yeah, okay. Goto the next one. (2) What is the correct answer to 0.7 ÷ 0.1?)

F) That’s nought point seventy five divided by nought point 1, ermmm.

T) What are you feeling now?

F) Okay, not confident, but…

T) Right. What’s making you not feel confident about it?

F) The whole kinda decimals, the nought in front of the, erm, second number.

T) Right, okay.

F) I think its that. (0.07 ×). (3) What is -3 x -2?) It’s minus three times minus two. I think we done quite a lot of this.

T) Right so you recognise it?

F) Three times two, minus six, that’s positive.

T) So, what made you decide it was plus six on that one? (+6 √)

F) Ermm coz we spent a lot of time on the box like minus and plus plus. [pause] (4) What is -3 - -2?) What is minus three minus minus two, ermm.

T) What about this one?

F) Not so sure because like all the minus’s together. It kinda confuses you.

T) Right so you’re feeling a bit of confusion?

F) Yep, so minus two one ermmm.

T) So what, how are you approaching?

F) I’m trying to remember the box, because of what we learnt.

T) Okay.

F) I think it’s that one [???] (+1 ×) I don’t get the question wrong.

T) Okay, (5) Which of the following is the same as 3/5?) so it’s an equivalent, which decimal there, is equivalent to three fifths?

F) Oh, I think its that (0.35 ×) I’m just having a guess.

T) Just having a guess?

F) Can’t remember.

T) So you’re just having a guess. What are your feelings, what are you feeling at that point?

F) Not entirely sure, ‘cause we haven’t done it, in quite a while, since last year.

T) Okay

F) (6) Which of the following is the same as 0.35?) [sv, “three, five, nought point three five”], thirty five nooo…

T) What are you feeling with this one?

F) Ermmm, okay (35/100 √), kind of a…

T) Are you confident about that or …?

F) Bit iffy.

T) … a bit uncertain or …

F) Bit uncertain, okay. (7) What is the answer to 31 – 17 (vertical arrangement)?) [sv “???”] oohh eaaasy, seventeen three twenty eleven twelve thirteen eugh [surprise] fourteen (14 √)

T) That was interesting, what were you feeling then when you, sudden, when you didn’t get what you were expecting?

F) It’s like ha, I thought it’d be there but it wouldn’t wasn’t, ‘cause it’s kinda a got something close to it

T) Then what did you do, you went to something that was close to it?

F) Yep.

T) Right, okay, [pause], did you do any sort of rechecking of ye?

F) I knew it would be kinda right, but it’s got to be something near it.

T) Okay. (8) What is the perimeter of this rectangle (7 by 5)?)

F) Er fourteen, twenty four (24 √)

T) Er okay what, what are you feeling with this one?

F) Erm quite easy ‘cause you add both the sides and the other two sides, we done quite a lot of work on that.

T) So there’s quite a lot of recognition?

F) Yeah, spent a lot of time, so its ooomm (9) Which is the correct formula for the circumference?) can’t remember this as much.

T) Okay have you ever seen it before?

F) We did it a bit last year, but it’s slightly confusing.

T) Okay

F) So it wasn’t like gone over dead thoroughly, I think it’s that (π x r2 ×), not sure.

T) Okay

F) [???] a go. (10) What is the median of the set of numbers 3, 7, 9, 2, 2) Ermm what’s the median, we didn’t spend a lot of time on this.

T) So what are you feeling then?

F) It’s quite confusing, it’s like all the different ones like range and everything.

T) So right you’re confused as…

F) Yeah, I get confused as to which one’s which.

T) Okay

T) [???]

F) erm ten, nineteen…

T) So which one are you gonna choose then?

F) Erm, add them all up and divide by how many there are.

T) Okay.

F) Uh, one two, erm int it the middle number I think, one two three four five, ermmm, aarrhh, I think its nine. (9 ×)

T) So what, what method did you use there?

F) Erm the two end numbers and the other two end numbers and the middle one, like for the median.

T) You looked for the middle one

F) Middle number, like for the median.

T) Oh right so, can you remember what it’s called when you add them up and divide by how many there are?

F) I think that might be the mean.

T) The mean, that’s right yeah. Er just click on finish now, thanks very much.

**Pupil G (Male)** Score : 5/10 (KS3 SATs Level : 6)

T) So what about that one? (1) What is the correct answer to 0.7 x 10?) [pause], what are you feeling when you see that?

H) Er, I feel as though I’ve seen it before … I know what to do, (7.0 √) yeah, click the answer

T) Yeah, yeah, click the answer and just go on to the next. (2) What is the correct answer to 0.7 ÷ 0.1?) [pause] What are you feeling with that one?

G) Bit confused.

T) Right, What’s confusing you?

G) ‘Cause, you know you’ve done it before but it was a long time ago and you’ve forgot how to do it. (0.07 ×)

T) What about that one then? (3) What is -3 x -2?)

G) That’s quite easy (+6 √)

T) Okay, so what are you feeling with that now?

G) Ermmm.

T) Now that you’ve selected an answer…

G) Bit relieved that I’ve finished the question.

T) Okay, click on the next one, question four , (4) What is -3 - -2?) minus three take away minus two. What are you feeling about that one?

G) It’s quite easy.

T) Quite easy? (-5 ×)

G) Yeah

T) No problems there?

G) No.

T) Question 5. (5) Which of the following is the same as 3/5?) Do you recognise these sorts of questions?

G) Yep.

T) What are you feeling, what, what’s going through your mind?

G) I’m thinking, er, what could it be, what’s close to it (0.35 ×) [long pause] [sv “ooh???”]

T) What are you feeling with that one? (6) Which of the following is the same as 0.35?)

G) Er, I feel as though I’ve seen it before, it’s, a hundreds, can I change the answer?

T) Yep, you can change the answer.

G) So

T) …decide to change? (35/100 √)

G) Coz, its, two digits after the decimal point so it must be hundred.

T) Okay, good. What about this one? (7) What is the answer to 31 – 17 (vertical arrangement)?) What are you feeling with this?

G) I’m thinking what sum? Got to do, ermm, what’s the easiest way to do it.

T) So you know what you’ve got to do?

G) Yeah.

T) So you’re just trying to think what’s the easiest?

G) Yeah

T) How confident are you that you can do this question?

G) Pretty confident. (14 √)

T) How are you feeling with this one? (8) What is the perimeter of this rectangle (7 by 5)?)

G) Thinking, er, how to, er, complete the question.

T) Okay what do you need to be able to do in order to be able to do that question?

G) Adding up and what the perimeter is, what the perimeter means. (24 √)

T) What about this question here? (9) Which is the correct formula for the circumference?) What are you feeling with this one?

G) Er quite confident, because we done it before. (π x r2 ×)

T) Right [very long pause] last one. (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

G) What is the median of a set of numbers?

T) What are you feeling with this one?

G) Erm, I’m trying to work out what the median means. (7 ×)

T) Right, so you’re not sure about that?

G) No

T) Okay, so what sort of emotion are you feeling?

G) Er, bit worried because I’m not sure what the answer could be.

T) Okay, [pause] why did you select two then?

G) ‘Cause it’s most often number.

T) Okay if I was to tell you that the median is the middle of the numbers when they’re in order what would you now feel?

G) Nine

T) Okay, but there was two parts to that wasn’t there? The middle of the numbers when they are in order.

G) So got to put them in order, [pause] so

T) Okay, that’ll do.

**Pupil H (Male)** Score : 7/10 (KS3 SATs Level : 6)

T) So there’s the first question (1) What is the correct answer to 0.7 x 10?) and er what are you thinking when you see that question?

H) Thinking er[???] zero point seven and I’ve got to ti it, got to times it by ten, so I move it up one, so then it’ll be seven point oh. (7.0 √)

T) How confident are you?

H) Very confident.

T) Okay go to the next one. (2) What is the correct answer to 0.7 ÷ 0.1?) Okay what are you thinking about this one then?

H) Got to divide it by nought point one ermm.

T) What are you feeling now?

H) Feeling like I got to use like, like bit scary if you get what I mean and you just like, don’t know what it is really. So I’ll just go, go for that. (0.70 ×)

T) So when you’re choosing that one is it like a guess or are you feeling confident?

H) Er this is a guess

T) Right, okay, next question. (3) What is -3 x -2?)

H) Well you know you’ve got your minus and your minus so you feel bit like you can do it but you can’t in one way.

T) Okay.

H) And then…

T) Have you ever seen multiplication of these sorts of numbers before?

H) Er yeah.

T) Er, so do you know some sort of, er have you got any knowledge on them?

H) Little bit yeah.

T) Okay.

H) Er quite confident with that one. (+6 √)

T) What about this one, question four? (4) What is -3 - -2?)

H) Er bit more complicated, ‘cause it’s got more minuses in, but I feel as if I can do it anyway (-5 ×)

T) [Very long pause] what about this one then? (5) Which of the following is the same as 3/5?), what are you feeling on this one?

H) Er you’ve got to know er quite comp, like complicated if you didn’t know what it is, er I’m okay with fractions so I’m quite confident about this one (0.6 √)

T) Okay, question six. (6) Which of the following is the same as 0.35?) What are you feeling with this one

H) Erm, basically, it’s which of the following, it’s like okay I can do it, and then you’ve just got to times it by a hundred, I’m quite confident with that. (35/100 √)

T) Right good (7) What is the answer to 31 – 17 (vertical arrangement)?)

H) Quite er, it’s not really too hard. (14 √)

T) … easy

H) Erm, well you can’t do one take away seven, so you got to go over to that one, your higher number, cross over, you’ve got a two there, and that, that comes eleven take away seven … which is four, four and then you’ve got to work out the rest.

T) Do you know any other ways of doing that?

H) Er, no, not really, it’s just the way I use.

T) Er okay, how confident are you?

H) Right okay, Quite confident with that. (8) What is the perimeter of this rectangle (7 by 5)?) [long pause] perimeter, I know I’ve gotta, I’m okay on these. Got to add all the sides [sv “fourteen, eh twenty [???]”] I’m quite confident on that one [sv “???”] erm it’s all right, you done a bit on these it’s like not long ago, er quite long ago, so you’re feeling like you might know what it is, but you might not so…

T) Okay

H) Er so go for that (24 √). (9) Which is the correct formula for the circumference?) Er its just like, it just kinda nearly remembered like, you’ve got your pi and you’ve got your times and I think it’s times two, ahhh, so I’m not too confident on that. (π x 2r √)

T) Last one. (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

H) Quite confident on this so. (9 ×)

T) What made you decide on nine

H) Well I looked at the three seven nine two two and I remembered the median is that I always remember it as the middle number, so then you go, you feel, you’ve got two on one side and then you see if you’ve got two on the other and your middle number’s the nine.

T) Okay, good, so yes that was right, that one wasn’t, Er that one you got right, well done. Er no, minus three and minus minus…

H) Yerh, comes a plus yeah

T) Become add two. Three fifths is nought point six, yes you said you were confident on that

H) Uh uh.

T) And you were confident on that one, You can always do it counting up as well.

H) Yeah.

T) To make twenty is three and another eleven…

H) Yer.

T) And you were right on your perimeter, and yes and you chose the right one there, a lot of people go for pi r squared.

H) Oh yeah

T) But you forgot to put them in order

H) Oh yeaaah.

T) What do you think about that, you said oh yeaaah?

H) Yeah I know [???] I forgot about that bit.

T) Very good.

**Pupil I (Male)** Score : 9/10 (KS3 SATs Level : 8)

T) There’s question one (1) What is the correct answer to 0.7 x 10?). So what are you feeling when you see that?

I) Okay nought point seven times ten, take the point over the seven, its seven point zero (7.0 √)

T) Okay so how do you feel about that question?

I) Slightly easy, but…

T) Are you feeling a bit confident about that one or slightly anxious or…?

I) Confident about it not(?)…

T) Next question (2) What is the correct answer to 0.7 ÷ 0.1?). What are you feeling about that one?

I) Right nought point one, eas, right you got to take the nought point one, change it into one, which make it times one, er and that’s, times ten, which is seven (7.0 √).

T) Okay.

I) Slightly bit more dubious, not sure about.

T) What’s causing you not to feel so sure about it?

I) Without the double negatives and the nought, dividing by the nought point one.

T) Okay. (3) What is -3 x -2?)

I) Minus three time minus two the minus two turn into divide it by two, so minus three divided by two is minus six and it’s slightly harder. (-6 ×)

T) And you’re feeling quite confident about it you say?

I) Yeah, slightly, not as confident as the one before. (4) What is -3 - -2?)

T) Okay, you seen these types of questions before?

I) Yeah, [sv “minus???”] minus one (-1 √), very confident about this one.

T) Right okay.

I) [???](5) Which of the following is the same as 3/5?) is the same as three fifths, right five and two hundreds is twenty and twenty times three is, nought point six (0.6 √), I’ve done a lot of these so.

T) That wasn’t with me was it?

I) It’s the following, (6) Which of the following is the same as 0.35?) thir, thirty five over a hundred (35/100 √).

T) So how are you feeling about that one? (7) What is the answer to 31 – 17 (vertical arrangement)?)

I) Easy, confident. What’s the answer to this calculation thirty one take seventeen, carry the one to make eleven. Eleven take away seven, eleven take away seven is four, two take away one is fourteen (14 √).

T) Okay, good, so you’re feeling pretty…

I) Pretty confident about that. (8) What is the perimeter of this rectangle (7 by 5)?) […] Seven time two and five times, seven times two is fourteen, plus five times two is ten, twenty four (24 √).

T) What made you decide to do it that way rather than any other way?

I) Just found it the easiest and the simplest.

T) Okay.

I) [sv “???”]

T) What are you feeling with that one? (9) Which is the correct formula for the circumference?)

I) Slightly confused by the perimeter and the area

T) Like so you know there’s two?

I) Like the circumference and the area, erm pi r squared is the area, so two pi r, I think, take that one (π x 2r √).

T) Okay.

I) Not very confident about that one ‘cause I get like confused a lot.

T) Your reason, your reasoning seemed okay.

I) I thought the squared would be for area, with it being squared, so, so it’s the only one that looked okay.

T) What about this? (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

I) It’s the median of these numbers, right put them in order, two, two, three, seven, nine, middle number out of the five is three (3 √).

T) Okay so you feel pretty confident about that?

I) Yeah.

T) So click on finish, Which ones did you get wrong, you got the minus three times minus two one wrong, minus times a minus?

I) Minus times a minus equals a plus, ahh did I put minus six

T) I don’t know what you put. I think you got confused with the division ones.

I) I just, got it.

T) Yeah, minus six, you did put minus six, yet you know the rule?

I) Yeah, I think that was just not looking at the signs, thinking it was too easy

T) Right thank you very much

**Pupil J (Female)** Score : 4/10 (KS3 SATs Level : 7)

T) Just get you on to the first question (1) What is the correct answer to 0.7 x 10?). So you tell me what you’re thinking, what do you think, what are you feeling at the moment?

J) Er, I think it’s a pretty [… ] easy question (7.0 √).

T) Select whichever you want [from the big list of emotion words] how confident are you feeling about that? On to the next one then (2) What is the correct answer to 0.7 ÷ 0.1?)

J) [Pause] er, it’s […] don’t really know how to do it.

T) What part of it is causing you the problem do you think?

J) It’s where you divide, I can’t, it’s just decimals, it’s just, I can’t deal with the decimals.

T) Okay have you done many of these before?

J) Er, not really.

T) Feeling a bit nervous now?

J) Yes (I don’t know how to do this).

T) You’re not going to be judged on this, it’s just for me to see what people are thinking you see?

J) (3) What is -3 x -2?) All minuses [verrrr – seems anxious?]

T) You don’t like negatives? Seen it before?

J) Yes.

T) Do you remember anything about it?

J) I remember a grid, but I can’t, I can’t, timesing it’s, er, I don’t know whether minus times minus equals a minus or not, ‘cause I know adding or subtracting but I don’t know…

T) Put down what you think. You feeling a bit anxious? (-6 ×) Was that a guess?

J) [Laughs nervously]

T) What about this one then? (4) What is -3 - -2?). What are you thinking about with this question then?

J) Um, I think, I think I know how to do it, but it’s, [pause] I’m not very confident about it.

T) Okay, What’s causing you the problem do you think, can you explain?

J) Er [pause] I don’t know.

T) Okay, well you just select whichever you want.

J) I think it’s minus one (-1 √).

T) How confident are you about that?

J) Not very [almost sighing]

T) (5) Which of the following is the same as 3/5?) You’re shaking your head?

J) [Laughs]

T) Why, what does that mean?

J) I can’t convert

T) You can’t convert?

J) It’s hard to convert it.

T) When you look at the answers, the responses there, what you’re thinking with that?

J) I don’t think its nought point three five, ‘cause I don’t think its got the same numbers in.

T) Right okay

J) Yearh [pause] they’re just jumbling the numbers round aren’t they [pause ] I don’t think it would, I don’t know why it would be that?

T) Nought point six?

J) erh

T) What’s going on with you you’re saying I don’t think it would be?

J) No, but, I don’t, I don’t see the resemblance or anything.

T) Okay, well whatever you think, or I just don’t know

J) I don’t know (I don’t know).

T) (6) Which of the following is the same as 0.35?) Or it’s going the other way round really [???]

J) Would it be that one?

T) Okay what makes you think that?

J) ‘Cause, that, err, No, ‘cause that would be 33 wouldn’t it, I mean thirty five [pause] out of a hundred.

T) That is thirty five out of a hundred!

J) Yeah, but, it would be … a decimal [pause] I think it would be that one cause they’re the hundreds aren’t they, or is it? Or is it that one?

T) Thirty five tenths?

J) I think its that one (35/10 ×).

T) Okay , how you feeling about this one? (7) What is the answer to 31 – 17 (vertical arrangement)?)

J) Ye, I take my time with it.

T) Okay, take your time then.

J) [Nervous laugh] [very long pause] fourteen. (14 √). (8) What is the perimeter of this rectangle (7 by 5)?) [Ohhh] perimeters.

T) What are you feeling with this one?

J) I get mixed up with perimeter and area, so I don’t, is it that, I think its them two times together.

T) Okay.

J) Which is thirty five (35 ×)

T) Okay, [pause](9) Which is the correct formula for the circumference?) You’re sighing quite a bit there?

J) [Nervous laugh].

T) What does that mean, how you feeling, are you finding it quite stressful?

J) Yeah, I know that, that, that is that an area?

T) Okay

J) Er yeah, no yeah [pause] no that a circumference.

T) Okay [pause] so how confident are you about that?

J) We did songs with Miss Hill so [pause] I’m pretty confident about that. (π x r2 ×)

T) Okay (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

J) Median […] median’s the middle number.

T) Okay.

J) [Laughs] I think, and that’s seven, [pause] so I think its seven.

T) When you say it’s the middle why have you chosen seven then?

J) Because you put them in order and seven will be the middle one then [pause] order [pause] two, two, three, oh no (raised pitch] it’s three [laughs]. (3 √)

T) Okay, just click on finish then, just have a look through. Ooh yeah a few of those decimals you didn’t get quite right.

J) Okay

T) Just have a look through.

J) Yeah [laugh]

T) Plus, minus times minus is plus

J) See [pause] correct one yay

T) Three fifths nought point six, one fifth is nought point two.

J) Okay.

T) You were right the first time going for thirty five over a hundred.

J) Oh no, [pause] I though it was because that’s in the tenths.

T) but that’s three tenths, see?

J) Yeah.

T) Can’t be thirty five tenths [pause] and yeah it’s the other way

J) And that’s the area, I knew that.

T) And you were right, that is the area.

J) Arhh nooo.

T) And you got that one right.

J) [Laughs].

T) Thank you very much, how are you feeling now? Glad its over?

J) Yeah, just a bit okay okay.

**Pupil K (Male)** Score : 7/10 (KS3 SATs Level : 7)

T) So there’s the first question (1) What is the correct answer to 0.7 x 10?), so what are you feeling when you look at that?

K) Er [pause] calm.

T) Calm, okay. You’re quite happy with that one? (7.0 √)

K) Yeah.

T) Seen this sort of question before? (2) What is the correct answer to 0.7 ÷ 0.1?)

K) Yep.

T) Yeah, okay, what are you feeling with that question?

K) Errr [very long pause] doubt.

T) Little bit of doubt?

K) Yeah.

T) Okay, what’s causing you to feel some doubt?

K) Not sure what the answer is.

T) Not sure what the answer is. What is it in the question that’s making you feel perhaps a bit doubtful?

K) Decimal.

T) Decimal, okay, alright.

K) [sv “???”]

T) So what are you doing, just guessing or?

K) It’s in the middle of it really. (0.07 ×)

T) Okay, what are you feeling with this question? (3) What is -3 x -2?)

K) Errr, confident.

T) Okay, so you’ve seen this before have you?

K) Yeah. (+6 √)

T) Okay. So question four (4) What is -3 - -2?), what about this one?

K) [Long pause] still confident.

T) Still confident, You’ve seen these sort of questions before have you?

K) Yeah.

T) Okay (+5 ×). (5) Which of the following is the same as 3/5?) [very long pause] (0.6 √) What were you feeling?

K) Satisfied.

T) Why were you feeling satisfied?

K) Know what the answer is.

T) You know what the answer is, right, cool.

K) It’s, yeah.

T) What are you feeling with that one? (6) Which of the following is the same as 0.35?)

K) Don’t really feel anything, feel anything.

T) Don’t feel anything?

K) No, (35/100 √) [pause] just [pause]

T) Okay, (7) What is the answer to 31 – 17 (vertical arrangement)?). What’s, what’s your instant feeling when you see that? [pause] do you recognise what sort of question it is?

K) Yeah.

T) So if you recognise it.

K) Er it’s (14 √)

T) How are you feeling about it now?

K) Eh sssss, calm really.

T) Calm?

K) Yeah.

T) Okay, do you think you got it right?

K) Yeah.

T) Yeah, good. What about this one, question 8? (8) What is the perimeter of this rectangle (7 by 5)?) (24 √) Okay what were you feeling when you did that. [pause] did you just knew it or?

K) Yeah.

T) What about this one? Do you recognise this sort of question? (9) Which is the correct formula for the circumference?) [pause] (π x r2 ×). What were you feeling on that one?

K) Doubt at first, but, just calm after.

T) Okay, what are you feeling with that one? (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

K) Unsure.

T) Unsure, what makes you feel unsure about it?

K) I can’t remember what median means.

T) Right, okay, So what are you going to decide?

K) Oh I can remember it now.

T) You can remember?

K) Yeah.

T) Okay, what is it then?

K) [Very long pause] it’s three. (3 √)

T) So what did you do for that one?

K) Put all the numbers in the correct order.

T) And then what?

K) Take the middle one.

T) Okay, right just press finish then… What are you feeling now at the end of the test?

K) Relief

T) Relief [laughs], okay thanks very much, very helpful.

**Pupil L (Female)** Score : 6/10 (KS3 SATs Level : 6)

T) I want you to tell me what you’re feeling and thinking about (1) What is the correct answer to 0.7 x 10?)

L) Right it’s er seven and it’s (7.0 √).

T) So, no problem with that one?

L) No that’s fine. Is it?

T) [Pause] So what are you feeling on that one? (2) What is the correct answer to 0.7 ÷ 0.1?)

L) Erm, I think it’s a bit harder, but it’s okay, is it [pause] that one? No.

T) What made you think praps it wasn’t?

L) Because, you’re dividing it by nought point one.

T) Right.

L) Erm, it’s got to be less it’s, is it that one? (0.70 ×)

T) I’m not saying.

L) Oh right.

T) I’m not saying whether you were right, wrong or anything.

L) Okay.

T) How confident were you on that one?

L) Not as confident as the first one.

T) What about question three? (3) What is -3 x -2?)

L) Right that’s minus six.

T) Okay

L) No six. (+6 √)

T) Okay what made you change your mind?

L) Because, I, I just didn’t look at the two minuses at first and then I realised.

T) Do you know something about two minuses?

L) Yeah, they make a plus.

T) Okay (4) What is -3 - -2?)

L) Ehmm (sv “make a plus”) erm two minuses make a plus so its minus three plus two is minus one. (-1 √)

T) Okay, How confident are you on that?

L) Er that one was fine. (5) Which of the following is the same as 3/5?) [sv “??? same as that”] erm, [pause] I’m not very good on fractions.

T) Okay

L) So it’s a bit harder.

T) Do you know you’re not very good or because you’ve seen that one and?

L) I’m, er, just, just not as good at fractions as other stuff.

T) Okay

L) Ermm, so, that, no, [pause] I’ll say that one. (0.35 ×)

T) How confident are you feeling?

L) Erm okay, not very confident though.

T) Okay, Do you think you got it right.

L) Yeah.

T) What about question 6? (6) Which of the following is the same as 0.35?)

L) [sv “which other one ???”], [pause] erm that one. (35/10 ×)

T) What about this question? What are you feeling with this question? (7) What is the answer to 31 – 17 (vertical arrangement)?)

L) Er that one’s okay [very long pause] er, that one take away that, erm, and then, so twenty six (26 ×)

T) Question eight. (8) What is the perimeter of this rectangle (7 by 5)?)

L) Er this one’s fine, I like stuff like this [sv “??? seven fourteen”] so it’s twenty four. (24 √)

T) So what made you decide to do it that way?

L) Erm, I just, I always, add the, like [pause] seven and seven, and then ‘cause it’s five and five and then add the two sevens and add the five.

T) [???] wasn’t a multiplication?

L) ‘Cause I know that perimeter is adding all the sides up.

T) Okay very good. What are you feeling on this one? (9) Which is the correct formula for the circumference?)

L) Erm, think I know it, it’s got [???] for the circumference, erm that’s area, erm no I think that one’s area though. (π x2r √)

T) Okay.

L) Erm, [long pause] is it, erm, I think the top one’s area actually though.

T) Right. Llast one. (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

L) [Very long pause] three. (3 √)

T) Okay, what made you decide on three?

L) I usually put them in order, then I sort of knock them down.

T) Okay so you knew what median was?

L) Yeah, finish?

T) Thank you very much.

**Pupil M (Female)** Score : 5/10 (KS3 SATs Level : 6)

T) So, what are you feeling on question one? (1) What is the correct answer to 0.7 x 10?)

M) Er, I’m not very good at multiplying decimals, erm, I think its nought point nought seven, (0.07 ×) but I’m not totally confident.

T) Okay.

M) So next?

T) Next question. (2) What is the correct answer to 0.7 ÷ 0.1?). Okay, what are you feeling when you see this one?

M) Erm, even less confident.

T) Less confident, why’s that?

M) [Laughs], yeah, it’s dividing decimals and it’s like, I don’t know. It looks harder, erm.

T) Have you ever seen anything like that before?

M) Yeah, but I’m in a classroom with other people, so it’s easier. Erm [sv “???”]. I’m just going to say nought point nought seven again. (0.07 ×) I’m not that confident at all.

T) Just guessing?

M) Yeah, it’s just a guess really. (3) What is -3 x -2?). What is minus three times minus two? Is it, I’m a bit more confident on this I think.

T) Okay, can you think of what’s coming into your mind as you think of that?

M) Just thinking, adding numbers together and [pause] then putting a minus (-6 ×). [Long pause] (4) What is -3 - -2?) Ermmm this one’s a bit harder, but I’m still okay with these [long pause] (+5 ×)

T) So you’re feeling quite confident?

M) Yeah.

T) Okay, question five. (5) Which of the following is the same as 3/5?)

M) [Long pause] Erm, I’m not too confident about fractions.

T) Right. Is there anything about fractions that make you feel unconfident?

M) Just don’t understand them really, ermm, I’m just gonna, I guess (0.35 ×)

T) Okay. Question 6 then. (6) Which of the following is the same as 0.35?)

M) Ermmm, I’m gonna, I think, I, I’m pretty certain on this, but I’m not completely certain.

T) You feel a bit more confident on this one?

M) Yeah on that one. (35/100 √) [long pause] (7) What is the answer to 31 – 17 (vertical arrangement)?) This one I’m quite confident on.

T) [???] subtraction?

M) Yeah.

T) What’s your method?

M) Erm, I’m gonna add, round the seventeen upto twenty, then take away, find the difference [pause] which is eleven and add the fir which is fourteen. (14 √)

T) Okay, good. So you’re pretty confident about that?

M) Yeah.

T) [???] (8) What is the perimeter of this rectangle (7 by 5)?)

M) Yeah, I’m okay with these just [sv “seven fourteen er”] twenty four. (24 √). I was confident on that. [Long pause] (9) Which is the correct formula for the circumference?) the circle.

T) What are you feeling now?

M) Erm, I think, Er I’m pretty [sv “five times”]

T) What feelings are you having?

M) Erm, a bit, I don’t know I jussst, I don’t know, I just trying to work it out.

T) You’re’ just [???] how to [???]

M) Yeah.

T) Have you seen circumference and area and stuff with circles before?

M) Yeah, I’m, we made a song up, so I’m trying to think of a song.

T) [Laughs]

M) Times pi by dee [sv ti] times pi by two then times by the radius, [pause], no I, can’t think of it.

T) Okay.

M) I’m just going to guess that it’s … pi times … two r (π x 2r √). Ahh) I’m just going to guess and ermm the median. (10) What is the median of the set of numbers 3, 7, 9, 2, 2?).

T) It was more than a guess wasn’t it?

M) That one, yeah I tried to work it out.

T) You kept hovering around that.

M) Yeah

T) Okay, so question ten.

M) Erm the middle number [sv, “two [pause], two, two, three, seven”] nine, I’m going to go for three. (3 √) I’m pretty sure on that, ‘cause median, middle number and I put them in order and it’s the middle number.

T) Okay

M) Press finish?

T) Yes, thank you. Feeling alright?

M) Yes. Fine.

**Pupil N (Male)** Score : 10/10 (KS3 SATs Level : 8+)

T) Tell me how you are feeling? (1) What is the correct answer to 0.7 x 10?)

N) I’m quite confident, because its not very difficult.

T) Do you just want to choose which one it is. (7.0 √) Okay. Question 2 (2) What is the correct answer to 0.7 ÷ 0.1?), what are you feeling?

N) Uhm, I had to think about it a bit more, ‘cause it’s the ride(?) one, uhmmm, its, I’m still pretty confident on what it is. (7.0 √)

T) What about this one? (3) What is -3 x -2?)

N) Yeah, I’m still pretty confident, because it’s not very difficult, and I remember that minus times a minus it’s positive (+6 √)

T) What about this one? (4) What is -3 - -2?)

N) I have, I have to think about ‘cause I can never get these, so I have to minus which is go down, then go up, so it’s minus minus two, which is minus one (-1 √).

T) What, what were you thinking in your mind?

N) I had to think minus, so you go, left, like on a number line.

T) Right, okay.

N) So left, but then it’s a minus a minus number so go right instead.

T) Okay, good. Question five then. (5) Which of the following is the same as 3/5?)

N) I just have ter, pretty confident and just have to remember which one it is so, a fifth is nought point two [sv “so three fifths is nought point six”]. (0.6 √)

T) Okay, good. (6) Which of the following is the same as 0.35?)

N) That’s dead easy, ‘cause its like percentage so [mumbles]. (35/100 √)

T) Tell me what you feel about this one (7) What is the answer to 31 – 17 (vertical arrangement)?)

N) A bit hard ‘cause its like got a drawing of it and so I just thirty one minus seventeen.

T) Okay. What [???] are you thinking of of solving it

N) Just the difference really, seventeen plus four is twenty one, then plus ten is thirty one, so fourteen. (14 √)

T) You didn’t try and mentally carry?

N) No. (8) What is the perimeter of this rectangle (7 by 5)?) Well you just add those two up and then double it, ‘cause it’s a perimeter.

T) Okay.

N) So its twenty, fer four (24 √)

T) What stopped you just multiplying the two numbers together?

N) ‘Cause that’s area and not perimeter.

T) So what are you thinking on this question? (9) Which is the correct formula for the circumference?)

N) I’m just, just looking at all, because that’s area and that’s just nothing, [sv “two two”] and that’s some weird thing and then that’s just the answer ‘cause it’s just remembering the formula.

T) Okay, so what formula did you remember?

N) Pi times dee, then I just had to think about it a bit, so it’s two r (π x 2r √)

T) And the last one? (10) What is the median of the set of numbers 3, 7, 9, 2, 2)

N) [SV “median”], I have to order them all, so its [sv “two two three”] so it’s two two three seven nine and then it’s. (3 √)

T) What stops you thinking it’s adding them all together and then?

N) ‘Cause that’s mean.

T) So you just know the word?

N) Yeh, so just finish now?

T) Yeah. How confident are you?

N) Pretty confident.

T) Okay, very good. What sort of feelings do you have, can you reflect on your feelings?

N) No, not really, not really kind a remembery kinda person

T) Right thanks very much.

**Pupil O (Male**) Score : 4/10 (KS3 SATs Level : 5) [This was the first one that I did as a trial, so some different questions and also had confidence selection turned on]

T) Looking at question 1 then. (1) What is the correct answer to 0.7 x 10?). What are you thinking what are your thoughts?

O) Er, confident.

T) You feeling fairly confident?

O) Yeah

T) Okay, is that strongly confident or strongly felt?

O) It’s er, okay.

T) Okay so do you want to select it then. (7.0 √)

O) It’s that isn’t it yep.

T) Yeah, you’ll have to tick the confident thing. So what about the next one? (2) What is the correct answer to 0.7 ÷ 0.1?)

O) [Very long pause] uneasy.

T) Yep, you’re not so sure about that one?

O) Doubtful.

T) Okay, which one are you going to choose then? [Very long pause] what are you thinking?

O) Deciding which one to do.

T) Have you done questions like that before?

O) Er These ones? Er no.

T) No, Any other words there that might strike you how you feel?

O) I’d er say a bit confused.

T) Okay, well you select whichever one you want (0.07 ×) [Long pause] on question 3, (3) What’s the value of 1 – 0.07?), what’s your immediate feeling?

O) Well, er, confident, errr, [long pause] unconcerned. (0.93 √)

T) You don’t have to use those words, whatever words you feel come up into your mind as well. What sort of thought processes are going on? You see questions like that before? (4) What is -3 x -2?)

O) Yep, er [very long pause].

T) What are you thinking?

O) Er, [blows], bit [???]stressed.

T) How are you feeling now, that as soon as you’ve plucked [sic] it? (+6 √)

O) Delight.

T) [Laughs] what ‘cause you’re finished? [laughs].

O) Yeah, out the way.

T) Okay what about this sort of question? (5) What is -3 - -2?), You seen these sort of questions before?

O) Yeah yeah, well I can say confident.

T) Highly confident in fact, select one then. (+5 ×) What are you feeling?

O) Confident.

T) Confident with question five. [Pause] (6) Which of the following is the same as 3/5?) How are you feeling about fractions and decimals?

O) Yeah, these, they’re okay better, fair, okay to do. (0.35 ×)

T) [Very long pause] See these questions before? Similar sort of question then, question seven. (7) Which of the following is the same as 0.35?). What sort of thoughts come into your mind as soon as you see nought point three five?

O) Well it’s the opposite of the other question.

T) Right. [Pause] (3/5 ×) You’re quite confident about that? [Long pause] (8) What’s the value of 0.1 x 0.1?). Again what sort of feelings you’re thinking?

O) Easy, straightforward. (0.1 ×)

T) Yep, okay, question nine (9) Which is larger 0.28 or 0.9?), which one’s larger?

O) Straightforward really. (0.9 √)

T) So you’re feeling quite good about that?

O) Uhm hum

T) About those questions, okay. [Pause] how are you going about solving question 10? (10) What is the answer to 31 – 17 (vertical arrangement)?) What are you doing mentally?

O) Well, take, like, it’s, er , like, answering, it’s answering the question.

T) Yep, [long pause]. (16 ×) How are you feeling about that one?

O) Feel alright, I’ve got er pressure on me.

T) What ‘cause I just asked you

O) Yeah.

T) Right, okay then [pause] hurry up then [laughs].

O) [laughs], [blows]

T) [Long pause], bit of uncertainty there perhaps, that, that’ll do then mate.

Notes: [sv ] means sub vocalisation, words the person was saying to themselves, but just audible.

[???] couldn’t decipher.

T is the teacher.

**Questions**

(1) What is the correct answer to 0.7 x 10?)

(2) What is the correct answer to 0.7 ÷ 0.1?)

(3) What is -3 x -2?)

(4) What is -3 - -2?)

(5) Which of the following is the same as 3/5?)

(6) Which of the following is the same as 0.35?)

(7) What is the answer to 31 – 17 (vertical arrangement)?)

(8) What is the perimeter of this rectangle (7 by 5)?)

(9) Which is the correct formula for the circumference?)

(10) What is the median of the set of numbers 3, 7, 9, 2, 2?)

**Appendix 6 Basic Pre and Post Test Results for Second Year 9 Cohort.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Misconceptions Remediation** | | |  | **Pre-Test** | |  | **Post Test** | |  |  |  |  |
| **Group** | **LName** | **FName** |  | **Score** | **%** |  | **Score** | **%** |  | **Mark Change** | **% Change** | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| A | Adams | Rebecca |  | 14 | 47 |  | 21 | 70 |  | **7** | **23** |  |
| A | Ashworth | Steven |  | 25 | 83 |  | 24 | 80 |  | **-1** | **-3** |  |
| A | Beacher | Olivia |  | 14 | 47 |  | 14 | 47 |  | **0** | **0** |  |
| A | Bell | Matthew |  | 20 | 67 |  | 21 | 70 |  | **1** | **3** |  |
| A | Chiang | Natalie |  | 17 | 57 |  | 19 | 63 |  | **2** | **7** |  |
| A | Cooke | Robyn |  | 15 | 50 |  | 19 | 63 |  | **4** | **13** |  |
| A | Curwen | Amelia |  | 22 | 73 |  | 25 | 83 |  | **3** | **10** |  |
| A | Drew | Thomas |  | 22 | 73 |  | 24 | 80 |  | **2** | **7** |  |
| A | Eardley | Joshua |  | 7 | 23 |  | 8 | 27 |  | **1** | **3** |  |
| A | Everall | Raymond |  | 11 | 37 |  | 12 | 40 |  | **1** | **3** |  |
| A | Fairhead | Amanda |  | 24 | 80 |  | 25 | 83 |  | **1** | **3** |  |
| A | Ferneyhough | Mark |  | 25 | 83 |  | 27 | 90 |  | **2** | **7** |  |
| A | Frost | Samantha |  | 7 | 23 |  | 6 | 20 |  | **-1** | **-3** |  |
| A | Graham | Hannah |  | 18 | 60 |  | 20 | 67 |  | **2** | **7** |  |
| A | Greaves | Georgia |  | 16 | 53 |  | 19 | 63 |  | **3** | **10** |  |
| A | Grzegorzek | Ashley |  | 7 | 23 |  | 9 | 30 |  | **2** | **7** |  |
| A | Hargreaves | Lee |  | 13 | 43 |  | 22 | 73 |  | **9** | **30** |  |
| A | Harley | Philippa |  | 17 | 57 |  | 21 | 70 |  | **4** | **13** |  |
| A | Harris | Freddy |  | 21 | 70 |  | 22 | 73 |  | **1** | **3** |  |
| A | Harrison | George |  | 14 | 47 |  | 18 | 60 |  | **4** | **13** |  |
| A | Holdcroft | Alexander |  | 21 | 70 |  | 23 | 77 |  | **2** | **7** |  |
| A | Holdcroft | Jack |  | 4 | 13 |  | 7 | 23 |  | **3** | **10** |  |
| A | Howard | Ashley |  | 16 | 53 |  | 17 | 57 |  | **1** | **3** |  |
| A | James | Thomas |  | 18 | 60 |  | 19 | 63 |  | **1** | **3** |  |
| A | Jamieson | Grant |  | 12 | 40 |  | 11 | 37 |  | **-1** | **-3** |  |
| A | Jones | Alex |  | 17 | 57 |  | 21 | 70 |  | **4** | **13** |  |
| A | Malpass | Sam |  | 14 | 47 |  | 19 | 63 |  | **5** | **17** |  |
| A | Morris | Daniel |  | 22 | 73 |  | 25 | 83 |  | **3** | **10** |  |
| A | Nagy | Lucy |  | 14 | 47 |  | 8 | 27 |  | **-6** | **-20** |  |
| A | Pointon | Alex |  | 17 | 57 |  | 20 | 67 |  | **3** | **10** |  |
| A | Proffitt | Jane |  | 21 | 70 |  | 25 | 83 |  | **4** | **13** |  |
| A | Proudmore | Thomas |  | 14 | 47 |  | 23 | 77 |  | **9** | **30** |  |
| A | Roberts | Kirsty |  | 11 | 37 |  | 19 | 63 |  | **8** | **27** |  |
| A | Robinson | Nathan |  | 6 | 20 |  | 9 | 30 |  | **3** | **10** |  |
| A | Rollison | Adam |  | 19 | 63 |  | 23 | 77 |  | **4** | **13** |  |
| A | Ryles | Maxwell |  | 13 | 43 |  | 18 | 60 |  | **5** | **17** |  |
| A | Sims | Lauren |  | 25 | 83 |  | 26 | 87 |  | **1** | **3** |  |
| A | Steele | Tara |  | 14 | 47 |  | 14 | 47 |  | **0** | **0** |  |
| A | Stone | Sophie |  | 13 | 43 |  | 19 | 63 |  | **6** | **20** |  |
| A | Stubbs | Matthew |  | 21 | 70 |  | 25 | 83 |  | **4** | **13** |  |
| A | Tudor | James |  | 15 | 50 |  | 20 | 67 |  | **5** | **17** |  |
| A | Walia | Rozie |  | 20 | 67 |  | 23 | 77 |  | **3** | **10** |  |
| A | Wint | Annabelle |  | 22 | 73 |  | 19 | 63 |  | **-3** | **-10** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **43** | **Averages** | **16.2** | **54.1** |  | **18.8** | **62.7** |  | **2.58** | **8.60** |  |
|  |  |  | **stddev** | **5.4** | **17.9** |  | **5.7** | **19.0** |  | **2.91** | **9.68** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| B | Arshad | Ebrahim |  | 29 | 97 |  | 30 | 100 |  | **1** | **3** |  |
| B | Baggaley | Amy |  | 21 | 70 |  | 20 | 67 |  | **-1** | **-3** |  |
| B | Bailey | Dean |  | 7 | 23 |  | 2 | 7 |  | **-5** | **-17** |  |
| B | Ball | Joshua |  | 21 | 70 |  | 22 | 73 |  | **1** | **3** |  |
| B | Bate | Oliver |  | 17 | 57 |  | 20 | 67 |  | **3** | **10** |  |
| B | Bayley | Joel |  | 22 | 73 |  | 19 | 63 |  | **-3** | **-10** |  |
| B | Belfield | James |  | 8 | 27 |  | 11 | 37 |  | **3** | **10** |  |
| B | Bruce | Joanne |  | 22 | 73 |  | 23 | 77 |  | **1** | **3** |  |
| B | Cooper | Christopher |  | 18 | 60 |  | 19 | 63 |  | **1** | **3** |  |
| B | Cooper | Danielle |  | 9 | 30 |  | 8 | 27 |  | **-1** | **-3** |  |
| B | Dale | Francesca |  | 15 | 50 |  | 13 | 43 |  | **-2** | **-7** |  |
| B | Durose | Ryan |  | 18 | 60 |  | 23 | 77 |  | **5** | **17** |  |
| B | Frost | Joanne |  | 12 | 40 |  | 12 | 40 |  | **0** | **0** |  |
| B | Gadd | Charles |  | 21 | 70 |  | 26 | 87 |  | **5** | **17** |  |
| B | Hackley | Michael |  | 22 | 73 |  | 22 | 73 |  | **0** | **0** |  |
| B | Hargreaves | William |  | 8 | 27 |  | 9 | 30 |  | **1** | **3** |  |
| B | Harold | Thomas |  | 22 | 73 |  | 22 | 73 |  | **0** | **0** |  |
| B | Hayward | Hannah |  | 27 | 90 |  | 25 | 83 |  | **-2** | **-7** |  |
| B | Henson | Catherine |  | 13 | 43 |  | 9 | 30 |  | **-4** | **-13** |  |
| B | Higginson | Michael |  | 13 | 43 |  | 14 | 47 |  | **1** | **3** |  |
| B | Holdcroft | Oliver |  | 9 | 30 |  | 9 | 30 |  | **0** | **0** |  |
| B | Huxley | Jordan |  | 18 | 60 |  | 20 | 67 |  | **2** | **7** |  |
| B | Irons | Chelsea |  | 20 | 67 |  | 25 | 83 |  | **5** | **17** |  |
| B | Johnson | Samuel |  | 13 | 43 |  | 20 | 67 |  | **7** | **23** |  |
| B | Lewis | Lauren |  | 15 | 50 |  | 16 | 53 |  | **1** | **3** |  |
| B | Machin | Alexander |  | 21 | 70 |  | 22 | 73 |  | **1** | **3** |  |
| B | Marshall | Dominic |  | 17 | 57 |  | 12 | 40 |  | **-5** | **-17** |  |
| B | Maurice | Thomas |  | 15 | 50 |  | 20 | 67 |  | **5** | **17** |  |
| B | McKeon | Tom |  | 15 | 50 |  | 23 | 77 |  | **8** | **27** |  |
| B | Merritt | Charnelle |  | 9 | 30 |  | 11 | 37 |  | **2** | **7** |  |
| B | Moore | Alice |  | 12 | 40 |  | 14 | 47 |  | **2** | **7** |  |
| B | Ogle | Michael |  | 11 | 37 |  | 8 | 27 |  | **-3** | **-10** |  |
| B | Richardson | Christopher |  | 23 | 77 |  | 27 | 90 |  | **4** | **13** |  |
| B | Roberts | Daniel |  | 13 | 43 |  | 24 | 80 |  | **11** | **37** |  |
| B | Robinson | Jacob |  | 24 | 80 |  | 26 | 87 |  | **2** | **7** |  |
| B | Rooke | Alexandra |  | 20 | 67 |  | 22 | 73 |  | **2** | **7** |  |
| B | Rowe | Nathan |  | 21 | 70 |  | 26 | 87 |  | **5** | **17** |  |
| B | Rowley | Dawn |  | 14 | 47 |  | 16 | 53 |  | **2** | **7** |  |
| B | Scott | Danielle |  | 19 | 63 |  | 19 | 63 |  | **0** | **0** |  |
| B | Simpson | Thomas |  | 19 | 63 |  | 22 | 73 |  | **3** | **10** |  |
| B | Slack | Daniel |  | 18 | 60 |  | 20 | 67 |  | **2** | **7** |  |
| B | Slinn | Matthew |  | 22 | 73 |  | 23 | 77 |  | **1** | **3** |  |
| B | Turley | Jade |  | 17 | 57 |  | 20 | 67 |  | **3** | **10** |  |
| B | Turnock | William |  | 16 | 53 |  | 19 | 63 |  | **3** | **10** |  |
| B | Ward | Michael |  | 15 | 50 |  | 16 | 53 |  | **1** | **3** |  |
| B | Williams | Holly |  | 10 | 33 |  | 13 | 43 |  | **3** | **10** |  |
| B | Williams | Owen |  | 18 | 60 |  | 21 | 70 |  | **3** | **10** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **47** | **Averages** | **16.79** | **56.0** |  | **18.36** | **61.2** |  | **1.57** | **5.25** |  |
|  |  |  | **stddev** | **5.20** | **17.3** |  | **6.16** | **20.5** |  | **3.14** | **10.47** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| C | Baddeley | Emily |  | 23 | 77 |  | 27 | 90 |  | **4** | **13** |  |
| C | Bailey | James |  | 19 | 63 |  | 24 | 80 |  | **5** | **17** |  |
| C | Barnett | Matthew |  | 13 | 43 |  | 25 | 83 |  | **12** | **40** |  |
| C | Beardmore | Jessica |  | 11 | 37 |  | 13 | 43 |  | **2** | **7** |  |
| C | Brereton | Ryan |  | 24 | 80 |  | 20 | 67 |  | **-4** | **-13** |  |
| C | Brock | Emma |  | 20 | 67 |  | 25 | 83 |  | **5** | **17** |  |
| C | Brown | Christopher |  | 16 | 53 |  | 20 | 67 |  | **4** | **13** |  |
| C | Caulfield | Megan |  | 25 | 83 |  | 23 | 77 |  | **-2** | **-7** |  |
| C | Clarke | Michael |  | 19 | 63 |  | 20 | 67 |  | **1** | **3** |  |
| C | Clarke | Stephanie |  | 10 | 33 |  | 18 | 60 |  | **8** | **27** |  |
| C | Condliffe | Lisa |  | 23 | 77 |  | 22 | 73 |  | **-1** | **-3** |  |
| C | Cooper | Matthew |  | 14 | 47 |  | 18 | 60 |  | **4** | **13** |  |
| C | Currie | Callum |  | 9 | 30 |  | 5 | 17 |  | **-4** | **-13** |  |
| C | Davies | Charlotte |  | 16 | 53 |  | 16 | 53 |  | **0** | **0** |  |
| C | Dawson | Jayson |  | 19 | 63 |  | 20 | 67 |  | **1** | **3** |  |
| C | Freakley | Emma |  | 18 | 60 |  | 18 | 60 |  | **0** | **0** |  |
| C | Gilpin | Victoria |  | 24 | 80 |  | 26 | 87 |  | **2** | **7** |  |
| C | Greaves | Shaun |  | 20 | 67 |  | 20 | 67 |  | **0** | **0** |  |
| C | Guest | George |  | 17 | 57 |  | 19 | 63 |  | **2** | **7** |  |
| C | Hayat | Zahra |  | 24 | 80 |  | 28 | 93 |  | **4** | **13** |  |
| C | Holdcroft | Carla |  | 7 | 23 |  | 7 | 23 |  | **0** | **0** |  |
| C | Holden | Yasmin |  | 8 | 27 |  | 16 | 53 |  | **8** | **27** |  |
| C | James | Emma |  | 15 | 50 |  | 17 | 57 |  | **2** | **7** |  |
| C | Jamieson | Sarah |  | 23 | 77 |  | 23 | 77 |  | **0** | **0** |  |
| C | Jenkins | Kentley |  | 28 | 93 |  | 27 | 90 |  | **-1** | **-3** |  |
| C | Johnson | Rosa |  | 26 | 87 |  | 28 | 93 |  | **2** | **7** |  |
| C | Johnston | Adam |  | 24 | 80 |  | 25 | 83 |  | **1** | **3** |  |
| C | Kent | Nicholas |  | 21 | 70 |  | 22 | 73 |  | **1** | **3** |  |
| C | Kettle | Stephanie |  | 17 | 57 |  | 21 | 70 |  | **4** | **13** |  |
| C | Lally | Kieran |  | 24 | 80 |  | 24 | 80 |  | **0** | **0** |  |
| C | Lingard | Adam |  | 2 | 7 |  | 4 | 13 |  | **2** | **7** |  |
| C | Mallin | Sean |  | 15 | 50 |  | 22 | 73 |  | **7** | **23** |  |
| C | Mcshee | James |  | 18 | 60 |  | 22 | 73 |  | **4** | **13** |  |
| C | Meir | Lucy |  | 10 | 33 |  | 17 | 57 |  | **7** | **23** |  |
| C | Nadin | Grace |  | 3 | 10 |  | 8 | 27 |  | **5** | **17** |  |
| C | O'Rourke | Harriet |  | 27 | 90 |  | 25 | 83 |  | **-2** | **-7** |  |
| C | Povey | Danielle |  | 17 | 57 |  | 19 | 63 |  | **2** | **7** |  |
| C | Prince | Jessica |  | 22 | 73 |  | 21 | 70 |  | **-1** | **-3** |  |
| C | Ridgway | Andrew |  | 23 | 77 |  | 22 | 73 |  | **-1** | **-3** |  |
| C | Robertson | Sean |  | 18 | 60 |  | 22 | 73 |  | **4** | **13** |  |
| C | Rudge | Lee |  | 11 | 37 |  | 12 | 40 |  | **1** | **3** |  |
| C | Sherratt | Natalie |  | 9 | 30 |  | 14 | 47 |  | **5** | **17** |  |
| C | Stanfield | Joe |  | 16 | 53 |  | 19 | 63 |  | **3** | **10** |  |
| C | Tatton | Valentina |  | 10 | 33 |  | 15 | 50 |  | **5** | **17** |  |
| C | Thomas | Matthew |  | 6 | 20 |  | 7 | 23 |  | **1** | **3** |  |
| C | Weston | Gemma |  | 16 | 53 |  | 19 | 63 |  | **3** | **10** |  |
| C | Worrall | Adam |  | 7 | 23 |  | 11 | 37 |  | **4** | **13** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **47** | **Averages** | **16.74** | **55.8** |  | **19.06** | **63.5** |  | **2.32** | **7.73** |  |
|  |  |  | **stddev** | **6.65** | **22.2** |  | **6.04** | **20.1** |  | **3.18** | **10.61** |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

This data only includes pupils who did both the pre and post tests.

**Appendix 7. Question Types used in CBA**

There are various types of question structure that can be posed by a CBA system. Generally there is a question stem which may contain a graphic or graphics, followed by some sort of response system. The more common question types are:

|  |  |
| --- | --- |
| Multiple Choice | A list of various options from which the test taker (participant) selects the option that they think is the answer to the question. This allows the use of options that might reveal the misconception. Such questions have to be carefully constructed to ensure that the distracter is in fact revealing the misconception. It is not always easy to construct such distractors (Lawson 1999, Harper 2003).  Pupils also have an unerring ability to think of possible answers that the question designer hadn’t even thought of, which might hide the true misconception, since the participant is not able to find an answer that reflects their true belief.  Multiple choice answers also have problems in revealing partially correct reasoning, especially if the question solution involves more than one step for solution.  Questions have been raised as to whether multiple choice actually allows the testing of deeper understanding, or does it simply provide assessment of lower levels of understanding (re Bloom’s Taxonomy) |
| Yes/No or True/False | Similar comments as above apply to this type of question structure as well, but it can become even more problematic in that the pupil may simply be guessing and happen upon the correct answer. |
| Multiple Response/Selection | In this type of question the participant is allowed to select more than one of the options. This can be quite useful in revealing conceptual understanding. For instance a statement could be made about a dataset in that all the values have been doubled. Options could then be provided making statements about what might have happened to the mean, median, inter-quartile range etc. Some might be true, some false. The participant has to look at each statement in turn and decide on its correctness (or not) (Lawson, 1999).  There are difficulties in providing a marking scheme for this type of question however and similar comments about guessing also apply. However if the system is able to provide detailed levels of response reporting, analysis may be able to reveal much. |
| Text Input/Text Match/Numeric Input/Response Entry/Free Response. | In these the participant has to enter the answer. This may be some word. For instance if the question stem was “what is the name of a five sided shape?” The participant would be expected to answer “Pentagon”. However even a simple question as this can result in problems. For instance what of the user said “regular pentagon”? Would the system be able to process this, should it be allowed anyway, or does the fact that they knew pentagon be given credit, or should the fact that they have a misconception on what “regular” means be identified?  What about spelling errors: pentigon, pentogone, pen ta gone?  The system needs to be capable of quite sophisticated syntactical parsing and possibly lexical analysis. The level of reporting can also be problematic and would require additional processing than that provided by a system such as Perception.  The response may also be the result of some calculation. Consider: “A man shares £5000 among his three children. How much will each get?”. So what should the participant input? 1666.66666667, 1666.66, 1666.67, £1666.67, £1666 and 67p, etc etc. There may be many possible correct answers, but the system has to be able to parse all these and make sense of them.  Even worse is in actually inputting and parsing algebraic expressions. There are a variety of methods to do this – some being more intuitive than others, but even then issues can arise with the equivalences between different representations of an algebraic expression, e.g. (1/2x or x/2 as a simple example (Strickland 2001, Beevers et al CALM project, Beevers 2003)  There is also the possibility of a wide variety of incorrect answers being given. Some of these may be amenable to analysis of why the answer is given, but some may not be (Neill 2000).  Of course when using a computer, the user may also have access to a calculator which may not be allowed in the context of the question! |
|  |  |

There are many other types of question as well (Twomey, Nichol & Smart 1999, Hodson, Saunders & Stubbs 2002, CAACentre), but only the more sophisticated systems will provide these features e.g. Perception (from QuestionMark). Often specialised, restricted availability software has to be purpose built.

Such additional question types include:

|  |  |
| --- | --- |
| Essay Grading | Much research is being done on this in the field of Artificial Intelligence, but the results, as might be expected, are far from encouraging, though apparently improving. A google search reveal many systems now being promoted. |
| Drag and Drop (Hotspot, Visual identification) | A graphic or label is positioned on a diagram or picture using drag and drop computer mouse technique. Sometimes it can be problematic for the software to recognise that the graphic is positioned correctly and is assessing what the user intended. |
| Selection/ Association | Items in one list are matched with items in another list. |
| Cloze (Missing word) | In this question type there are gaps in a statement, sentence, or mathematical argument etc and the user attempts to determine what the missing object might be to enable the statement to make sense (Dale, 1999) |
| Sore finger | As in “sticks out like a sore finger” . This has been used in language teaching and also in teaching computer programming. The user tries to pick out a word or item that is inappropriate to the context. |
| Ranking, sequencing | The user orders the possible answer items according to some given criteria. |
| Assertion/Reason | An assertion is made and the test taker must select from a list of reasons which they think most closely matches. There are variations on this theme, such as determining if a given reason fits a given assertion. Although it is slightly more complex the evidence is that students soon get used to it (Lawson 1999). One of the drawbacks of this method is that it does often require a high level of linguistic skill and so it is recommended for formative use only (Williams 2001). One study indicates that performance on assertion/reason is lower than for other types of question format (Skakun et al 1979). However it does offer more potential in testing the higher level cognitive skills. |
| Find the Exception | This is a variation of multiple choice in which all the options have to be considered and identified as being correct or incorrect in order to find the incorrect response (UCD Centre <http://www.ucd.ie/teaching/assess/as10.htm>, Lo, Wang & Yeh 2003) |
| Best Answer | There may not be a “correct” answer, but the student has to select the best answer available – again linguistic issues arise here. |
| Free Choice | Students select as many options as they need in order to be certain of getting one of them correct. The more they choose the less marks available. This method attempts to quantify certainty of knowledge (Bruce, Private Correspondence). |
| Grid Based Scheme | An innovative method being developed by Thomas, Oktun and Buis (2002) whereby a student scans a 3x3 grid of cells containing information, looking for those that “collectively constitute a correct and complete response” to a given task e.g. looking for equivalent percentages, decimals and fractions, or that may require the ordering of the cells according to some criterion. Hopefully this method aids the assessment of higher order skills. |
| Confidence Based Marking (CBM) | With this method students have to provide an indication of how confident they are about the answer. See literature review. |

**Issues other than identified in the literature review.**

One of the clearly identified drawbacks of many of these question types, especially with Multiple Choice, is that of the effect of students simply guessing the answer (Lawson 1999), which does not reveal the truth of the misconception.

Bush (1999) shows that to get 40% on a MC test with 4 options per question, the student only needs to actually know the correct answer to 20% of the questions. Guessing on the remaining 80% will (probably) result in the other 20%! Bush discusses various ways that assigning marks can be used to offset the effect of random guessing

There is also an associated issue with pupils not actually carrying out any mathematical calculations. I have even observed, on more than one occasion, a sort of random game playing mentality, where, in particular lower ability students, simply have a random strategy of selection, with no seemingly conscious thought being applied. This may be occurring unobserved among other, even more able pupils.

Thompson, Beckmann and Senks (1997) expressed the concern that teachers and pupils might have difficulties in understanding the mark schemes used.

It can be useful if the user has to work out the answer for themselves and enter the answer freely. However there are syntactical processing difficulties with this, but Newble, Baxter and Elmslie (1979) noted that free choice/response or text input gave a lower mark than using multiple choice.

Students suggested that the free response format allowed them to demonstrate their (clinical) skill more clearly. Hawkes (1998) also noted that MC gave students little practice in expressing their mathematical thoughts or in using symbolism and symbolic methods.

In mathematics there may be a number of steps required to get a solution and any of these steps can introduce errors (Lawson 1999, Hawkes 1998). Students may not be able to break down a solution into the multi steps required or may simply stop after the first step (a delta 2 failure) (Smit, Oosterhout and & 1996, Lawson 1999, Kuppermintz et al 1999).

CBA software rarely takes into account this issue or in giving partial credits for parts of an answer that are correct or have been correctly worked out, though with the wrong numbers (method marks or follow through). There are some notable exceptions to this (Beevers 2003, Beevers et al CALM project). Some software can also provide clues as required by a user in working through a problem and penalise accordingly (Beevers et al CALM project, Lawson 1999).

Some systems allow randomisation of question order and item order and some even allow the randomisation of the values in a question as well (Bush 1999, Hunt - CAMPUS, Greenhow - Mathletics, Thelwall - Wolverhampton, Beevers -Scholar etc). It has been reported by Clariana & Wallace (2002) that the order of questions and the order of responses can have an effect on performance. Greenhow (2002) reports that the use of non-invigilated and repeatable on-demand tests do not appear to rank students correctly.

Some of the other issues related to the use of CBA include the motivational aspect of CBA. It can certainly be more entertaining (Hodson, Saunders & Stubbs, 2002), there is often novelty value (Fuson & Brinko 1985) and Ketamo & Multisilta (2003) note that digital learning materials seem to motivate pupils more than traditional materials. Others comment that the conversion of paper and pencil tests to CBA often fails to utilise multimedia effectively or at all (Bull 1999). Lo, Wang & Yeh (2003) made comments on the possibility of learner disorientation from poor navigational controls and on cognitive overload.

Some major studies seem to cast doubt on the assumed benefits of using computer technology (Roberts & Stephens 1999, Kerawella & Crook 2002, Angrist & Lavy 2002, The Economist 2002, McDougall 2001). Whereas a meta-analysis by Christmann & Badgett (1999) indicated that using computer aided instruction (CAI) had a higher achievement, being most effective with students in urban areas and less with this from rural areas. Clariana & Wallace (2002) noted that higher attaining students seemed to benefit more from CBA, but this could be due to other factors such as a propensity to work hard anyway (Wong, Wong & Yeung 2001). Yet students feel that some CBA such as CBM does help them identify areas of weakness that they have (Gardner-Medwin 2004).

Contrasting the entertainment value of CBA is the fact that multimedia can be distracting (Kyle 1999). There are also identified gender issues to be considered with multimedia and how boys and girls respond to different designs (Passig & Levin 2000, Inkpen et al 1994, Lawry et al 1995)

The physical process of actually being a student taking a CBA test can also introduce concerns. Tiredness is an issue reported on by several researchers (Oliver 2000, Twomey, Nicol & Smart 1999, Clariana & Wallace 2002). Often this was due to low screen resolutions, but this is becoming less of an issue nowadays. It is recommended however that on-line tests should not last longer than 30-45 minutes (Bull, McKenna 2001). It has also been noted that it takes longer to read information from a screen (Oliver 2000, Clariana & Wallace 2000, Ketamo & Multisilta 2003). This may be a factor affecting validity if the student misreads or misinterprets the information. (Also images which may scale improperly (me)). Some students do exhibit computer phobia (Hodson, Saunders & Stubbs 2002, Bull 1999). CBA does often require students to be properly trained in its use, though often this does not take too long, however some can “slip through the net” in post pilot usage (Hodson et al 2002).

Anxiety may be increased when taking a computer based test because of the novelty of the situation, technophobia and the anxiety caused by a student wondering if a computer based test accurately reflects their knowledge anyway (perhaps because of marking issues) (Beevers et al CALM Project). Hancock (2001) identified a number of issues associated with test anxiety and noted that greater “evaluative threat” can lead to poorer performance. Bull & Stephens (1999) suggest from their experiments in Luton that anxiety levels may be reduced by using CBA.

In contrast to this is the fact that students can often feel under less threat because they are dealing with a machine that doesn’t judge them or because they feel less embarrassed (Lawson 1999). They also find it less threatening that they can change an answer at will, without making a mess of the paper – a real issue for some, especially for girls perhaps (personal anecdotal observation) (Bull & Stephens 1999). There is also the factor that any marker predisposition is removed (Oliver 2000).

A key issue in using CBA is that the computer doesn’t get bored (Pellone 1991). It can be used by a student over and over and won’t get frustrated with the user. This means that a student can keep on using the program as often as he or she wishes. Students may not feel inclined to use the program though if there is no perceived reward or value in doing so (Hodson et al 2002).

It is often suggested as an argument against using CBA and multiple choice in particular is that such systems only allow low order thinking skills to be tested. In terms of Bloom’s Taxonomy of Educational Objectives (TAL) this relates to knowledge and comprehension and perhaps Analysis. Many have however been surprised that CBA can test higher level understanding if careful questions are designed (Hawkes 1998, Bush 1999). Some also feel that the use of MC leads to more use of surface level learning strategies or use of different strategies than with other assessment media (Steffanou & Parkes 2003), but Bull & Stephens (1999) noted that students with a propensity to deep learning styles continue to use such strategies regardless of the test instrument.

Hellstrom, Lindstrom & Wastle (undated) reported that when trying to classify a large test bank of questions it was actually quite difficult to classify what type of thinking a particular question employs.

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**Appendix 8 Confidence Rating per Pupil (1st Cohort Y9) by Pupil Test Score**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Confidence Scores by Pupil for each correct and wrong answer** | | | | | | | | | | | | | | | | |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | **Correct** | | |  |  |  |  |  | **Incorrect** | | | |  |  |  |  | **Total** | |  |  |  |  |  |
| **Pupil No.** | | **0** | **1** | **2** | **3** |  | **Avg Conf** | **Count** | | **0** | **1** | **2** | **3** |  | **Avg Conf** | **Count** | | **0** | **1** | **2** | **3** |  | **Avg Conf** | **Score** |
| 3 |  |  | 2 | 1 | 24 |  | **2.8** | **27** |  |  |  | 1 | 2 |  | **2.7** | **3** |  | 0 | 2 | 2 | 26 |  | **2.8** | **27** |
| 23 |  |  | 6 | 8 | 13 |  | **2.3** | **27** |  |  |  | 3 |  |  | **2.0** | **3** |  | 0 | 6 | 11 | 13 |  | **2.2** | **27** |
| 25 |  |  | 2 | 4 | 21 |  | **2.7** | **27** |  |  |  | 1 | 2 |  | **2.7** | **3** |  | 0 | 2 | 5 | 23 |  | **2.7** | **27** |
| 116 |  |  |  | 1 | 26 |  | **3.0** | **27** |  |  |  | 1 | 2 |  | **2.7** | **3** |  | 0 | 0 | 2 | 28 |  | **2.9** | **27** |
| 32 |  |  |  | 7 | 19 |  | **2.7** | **26** |  |  |  | 2 | 2 |  | **2.5** | **4** |  | 0 | 0 | 9 | 21 |  | **2.7** | **26** |
| 18 |  |  |  | 3 | 22 |  | **2.9** | **25** |  |  | 1 |  | 4 |  | **2.6** | **5** |  | 0 | 1 | 3 | 26 |  | **2.8** | **25** |
| 22 |  |  | 1 | 6 | 18 |  | **2.7** | **25** |  |  |  | 2 | 3 |  | **2.6** | **5** |  | 0 | 1 | 8 | 21 |  | **2.7** | **25** |
| 68 |  |  |  | 1 | 24 |  | **3.0** | **25** |  |  |  |  | 5 |  | **3.0** | **5** |  | 0 | 0 | 1 | 29 |  | **3.0** | **25** |
| 69 |  |  | 2 | 7 | 16 |  | **2.6** | **25** |  |  | 1 |  | 4 |  | **2.6** | **5** |  | 0 | 3 | 7 | 20 |  | **2.6** | **25** |
| 75 |  |  |  | 6 | 19 |  | **2.8** | **25** |  |  | 1 | 1 | 3 |  | **2.4** | **5** |  | 0 | 1 | 7 | 22 |  | **2.7** | **25** |
| 103 |  |  |  |  | 25 |  | **3.0** | **25** |  |  |  | 1 | 4 |  | **2.8** | **5** |  | 0 | 0 | 1 | 29 |  | **3.0** | **25** |
| 13 |  |  | 2 | 1 | 21 |  | **2.8** | **24** |  |  |  | 2 | 4 |  | **2.7** | **6** |  | 0 | 2 | 3 | 25 |  | **2.8** | **24** |
| 15 |  |  |  | 1 | 23 |  | **3.0** | **24** |  |  |  | 1 | 5 |  | **2.8** | **6** |  | 0 | 0 | 2 | 28 |  | **2.9** | **24** |
| 26 |  |  |  |  | 24 |  | **3.0** | **24** |  |  |  | 1 | 5 |  | **2.8** | **6** |  | 0 | 0 | 1 | 29 |  | **3.0** | **24** |
| 54 |  |  |  | 2 | 22 |  | **2.9** | **24** |  |  |  | 1 | 4 |  | **2.8** | **5** |  | 0 | 0 | 3 | 26 |  | **2.8** | **24** |
| 63 |  |  | 2 | 4 | 18 |  | **2.7** | **24** |  |  |  | 1 | 4 |  | **2.8** | **5** |  | 0 | 2 | 5 | 22 |  | **2.6** | **24** |
| 64 |  |  | 2 | 5 | 17 |  | **2.6** | **24** |  | 1 | 1 | 1 | 3 |  | **2.0** | **6** |  | 1 | 3 | 6 | 20 |  | **2.5** | **24** |
| 77 |  |  |  | 1 | 23 |  | **3.0** | **24** |  |  |  | 1 | 5 |  | **2.8** | **6** |  | 0 | 0 | 2 | 28 |  | **2.9** | **24** |
| 6 |  |  |  | 3 | 20 |  | **2.9** | **23** |  |  | 1 |  | 6 |  | **2.7** | **7** |  | 0 | 1 | 3 | 26 |  | **2.8** | **23** |
| 41 |  |  | 3 | 5 | 15 |  | **2.5** | **23** |  |  | 2 | 2 | 3 |  | **2.1** | **7** |  | 0 | 5 | 7 | 18 |  | **2.4** | **23** |
| 58 |  |  | 1 | 13 | 9 |  | **2.3** | **23** |  |  | 2 | 3 | 1 |  | **1.8** | **6** |  | 0 | 3 | 16 | 10 |  | **2.2** | **23** |
| 65 |  |  |  | 6 | 17 |  | **2.7** | **23** |  |  | 1 | 3 | 3 |  | **2.3** | **7** |  | 0 | 1 | 9 | 20 |  | **2.6** | **23** |
| 83 |  |  |  | 13 | 10 |  | **2.4** | **23** |  |  |  | 7 |  |  | **2.0** | **7** |  | 0 | 0 | 20 | 10 |  | **2.3** | **23** |
| 104 |  |  |  |  | 23 |  | **3.0** | **23** |  |  |  |  | 7 |  | **3.0** | **7** |  | 0 | 0 | 0 | 30 |  | **3.0** | **23** |
| 7 |  | 1 | 2 | 4 | 15 |  | **2.5** | **22** |  |  | 2 | 2 | 4 |  | **2.3** | **8** |  | 1 | 4 | 6 | 19 |  | **2.4** | **22** |
| 9 |  | 1 | 1 | 7 | 13 |  | **2.5** | **22** |  | 1 | 3 | 2 | 2 |  | **1.6** | **8** |  | 2 | 4 | 9 | 15 |  | **2.2** | **22** |
| 10 |  |  | 2 | 8 | 12 |  | **2.5** | **22** |  |  | 1 | 4 | 3 |  | **2.3** | **8** |  | 0 | 3 | 12 | 15 |  | **2.4** | **22** |
| 29 |  |  |  | 4 | 18 |  | **2.8** | **22** |  |  |  | 3 | 5 |  | **2.6** | **8** |  | 0 | 0 | 7 | 23 |  | **2.8** | **22** |
| 38 |  |  | 2 | 9 | 11 |  | **2.4** | **22** |  |  | 3 | 2 | 3 |  | **2.0** | **8** |  | 0 | 5 | 11 | 14 |  | **2.3** | **22** |
| 53 |  |  |  | 2 | 20 |  | **2.9** | **22** |  |  |  |  | 8 |  | **3.0** | **8** |  | 0 | 0 | 2 | 28 |  | **2.9** | **22** |
| 90 |  |  |  | 7 | 15 |  | **2.7** | **22** |  |  |  | 4 | 4 |  | **2.5** | **8** |  | 0 | 0 | 11 | 19 |  | **2.6** | **22** |
| 95 |  |  |  | 1 | 21 |  | **3.0** | **22** |  |  |  | 1 | 7 |  | **2.9** | **8** |  | 0 | 0 | 2 | 28 |  | **2.9** | **22** |
| 44 |  |  |  | 1 | 20 |  | **3.0** | **21** |  |  | 3 |  | 6 |  | **2.3** | **9** |  | 0 | 3 | 1 | 26 |  | **2.8** | **21** |
| 56 |  |  |  | 2 | 19 |  | **2.9** | **21** |  |  |  | 1 | 8 |  | **2.9** | **9** |  | 0 | 0 | 3 | 27 |  | **2.9** | **21** |
| 85 |  |  |  | 4 | 17 |  | **2.8** | **21** |  |  |  | 1 | 8 |  | **2.9** | **9** |  | 0 | 0 | 5 | 25 |  | **2.8** | **21** |
| 87 |  |  |  | 2 | 19 |  | **2.9** | **21** |  |  | 1 | 3 | 5 |  | **2.4** | **9** |  | 0 | 1 | 5 | 24 |  | **2.8** | **21** |
| 94 |  |  | 2 | 6 | 13 |  | **2.5** | **21** |  |  | 1 | 4 | 4 |  | **2.3** | **9** |  | 0 | 3 | 10 | 17 |  | **2.5** | **21** |
| 99 |  |  | 1 | 3 | 17 |  | **2.8** | **21** |  |  | 2 | 2 | 5 |  | **2.3** | **9** |  | 0 | 3 | 5 | 22 |  | **2.6** | **21** |
| 101 |  |  |  | 3 | 18 |  | **2.9** | **21** |  |  |  | 4 | 5 |  | **2.6** | **9** |  | 0 | 0 | 7 | 23 |  | **2.8** | **21** |
| 20 |  |  | 2 | 16 | 2 |  | **2.0** | **20** |  |  | 1 | 7 | 2 |  | **2.1** | **10** |  | 0 | 3 | 23 | 4 |  | **2.0** | **20** |
| 28 |  |  |  | 5 | 15 |  | **2.8** | **20** |  |  | 2 | 2 | 6 |  | **2.4** | **10** |  | 0 | 2 | 7 | 21 |  | **2.6** | **20** |
| 35 |  | 1 | 3 | 2 | 14 |  | **2.5** | **20** |  |  | 3 | 4 | 3 |  | **2.0** | **10** |  | 1 | 6 | 6 | 17 |  | **2.3** | **20** |
| 51 |  |  |  | 7 | 13 |  | **2.7** | **20** |  |  |  | 8 | 2 |  | **2.2** | **10** |  | 0 | 0 | 15 | 15 |  | **2.5** | **20** |
| 60 |  |  | 1 | 5 | 14 |  | **2.7** | **20** |  |  |  | 6 | 4 |  | **2.4** | **10** |  | 0 | 1 | 11 | 18 |  | **2.6** | **20** |
| 74 |  |  |  | 1 | 19 |  | **3.0** | **20** |  |  |  |  | 10 |  | **3.0** | **10** |  | 0 | 0 | 1 | 29 |  | **3.0** | **20** |
| 19 |  |  |  | 4 | 15 |  | **2.8** | **19** |  | 1 | 1 | 3 | 4 |  | **2.1** | **9** |  | 1 | 1 | 7 | 19 |  | **2.4** | **19** |
| 70 |  |  |  | 6 | 13 |  | **2.7** | **19** |  |  |  | 4 | 7 |  | **2.6** | **11** |  | 0 | 0 | 10 | 20 |  | **2.7** | **19** |
| 72 |  |  |  |  | 19 |  | **3.0** | **19** |  |  |  |  | 11 |  | **3.0** | **11** |  | 0 | 0 | 0 | 30 |  | **3.0** | **19** |
| 82 |  |  | 1 | 5 | 13 |  | **2.6** | **19** |  |  |  | 8 | 3 |  | **2.3** | **11** |  | 0 | 1 | 13 | 16 |  | **2.5** | **19** |
| 98 |  |  |  | 5 | 14 |  | **2.7** | **19** |  |  | 1 | 2 | 8 |  | **2.6** | **11** |  | 0 | 1 | 7 | 22 |  | **2.7** | **19** |
| 115 |  |  | 3 | 13 | 3 |  | **2.0** | **19** |  |  | 2 | 8 | 1 |  | **1.9** | **11** |  | 0 | 5 | 21 | 4 |  | **2.0** | **19** |
| 21 |  |  |  | 11 | 7 |  | **2.4** | **18** |  |  | 1 | 5 | 6 |  | **2.4** | **12** |  | 0 | 1 | 16 | 13 |  | **2.4** | **18** |
| 43 |  | 1 | 7 | 3 | 7 |  | **1.9** | **18** |  | 1 | 5 | 3 | 2 |  | **1.5** | **11** |  | 2 | 12 | 6 | 9 |  | **1.7** | **18** |
| 71 |  |  |  | 3 | 15 |  | **2.8** | **18** |  |  | 2 | 4 | 6 |  | **2.3** | **12** |  | 0 | 2 | 7 | 21 |  | **2.6** | **18** |
| 84 |  |  | 5 | 4 | 9 |  | **2.2** | **18** |  | 1 | 1 | 4 | 6 |  | **2.3** | **12** |  | 1 | 6 | 8 | 15 |  | **2.2** | **18** |
| 96 |  |  |  | 2 | 16 |  | **2.9** | **18** |  |  |  | 1 | 10 |  | **2.9** | **11** |  | 0 | 0 | 3 | 26 |  | **2.8** | **18** |
| 8 |  |  | 1 | 3 | 13 |  | **2.7** | **17** |  | 1 | 1 | 3 | 6 |  | **2.3** | **11** |  | 1 | 2 | 6 | 19 |  | **2.4** | **17** |
| 24 |  |  | 4 | 2 | 11 |  | **2.4** | **17** |  |  | 5 | 2 | 2 |  | **1.7** | **9** |  | 0 | 9 | 4 | 13 |  | **1.9** | **17** |
| 46 |  |  | 1 | 3 | 13 |  | **2.7** | **17** |  | 1 | 1 | 4 | 6 |  | **2.3** | **12** |  | 1 | 2 | 7 | 19 |  | **2.4** | **17** |
| 48 |  |  |  |  | 17 |  | **3.0** | **17** |  |  |  |  | 13 |  | **3.0** | **13** |  | 0 | 0 | 0 | 30 |  | **3.0** | **17** |
| 78 |  | 1 | 1 | 11 | 4 |  | **2.1** | **17** |  | 3 | 2 | 6 | 2 |  | **1.5** | **13** |  | 4 | 3 | 17 | 6 |  | **1.8** | **17** |
| 102 |  |  |  | 5 | 12 |  | **2.7** | **17** |  |  | 2 | 8 | 3 |  | **2.1** | **13** |  | 0 | 2 | 13 | 15 |  | **2.4** | **17** |
| 106 |  |  | 2 | 3 | 12 |  | **2.6** | **17** |  |  | 5 | 2 | 6 |  | **2.1** | **13** |  | 0 | 7 | 5 | 18 |  | **2.4** | **17** |
| 107 |  |  | 1 | 4 | 12 |  | **2.6** | **17** |  |  | 2 | 5 | 6 |  | **2.3** | **13** |  | 0 | 3 | 9 | 18 |  | **2.5** | **17** |
| 109 |  |  | 1 | 5 | 11 |  | **2.6** | **17** |  |  | 2 | 2 | 9 |  | **2.5** | **13** |  | 0 | 3 | 7 | 20 |  | **2.6** | **17** |
| 112 |  |  | 1 | 1 | 15 |  | **2.8** | **17** |  | 1 |  | 2 | 10 |  | **2.6** | **13** |  | 1 | 1 | 3 | 25 |  | **2.7** | **17** |
| 79 |  | 1 | 2 | 4 | 9 |  | **2.3** | **16** |  |  | 1 | 5 | 8 |  | **2.5** | **14** |  | 1 | 3 | 9 | 17 |  | **2.4** | **16** |
| 36 |  |  | 1 | 6 | 8 |  | **2.5** | **15** |  |  | 1 | 10 | 4 |  | **2.2** | **15** |  | 0 | 2 | 16 | 12 |  | **2.3** | **15** |
| 59 |  |  | 2 | 1 | 12 |  | **2.7** | **15** |  | 3 |  | 2 | 10 |  | **2.3** | **15** |  | 3 | 2 | 3 | 22 |  | **2.5** | **15** |
| 92 |  |  | 4 | 3 | 8 |  | **2.3** | **15** |  | 3 | 4 | 4 | 4 |  | **1.6** | **15** |  | 3 | 8 | 7 | 12 |  | **1.9** | **15** |
| 11 |  | 2 | 3 | 6 | 3 |  | **1.7** | **14** |  | 1 | 8 | 4 | 2 |  | **1.5** | **15** |  | 3 | 11 | 10 | 5 |  | **1.5** | **14** |
| 39 |  |  | 2 | 7 | 5 |  | **2.2** | **14** |  |  | 5 | 5 | 6 |  | **2.1** | **16** |  | 0 | 7 | 12 | 11 |  | **2.1** | **14** |
| 49 |  |  | 2 | 5 | 7 |  | **2.4** | **14** |  | 2 | 4 | 6 | 2 |  | **1.6** | **14** |  | 2 | 6 | 11 | 9 |  | **1.8** | **14** |
| 73 |  |  | 5 | 4 | 5 |  | **2.0** | **14** |  | 1 | 6 | 3 | 6 |  | **1.9** | **16** |  | 1 | 11 | 7 | 11 |  | **1.9** | **14** |
| 14 |  | 1 | 2 | 3 | 7 |  | **2.2** | **13** |  | 1 | 5 | 5 | 6 |  | **1.9** | **17** |  | 2 | 7 | 8 | 13 |  | **2.1** | **13** |
| 31 |  | 2 | 3 | 1 | 7 |  | **2.0** | **13** |  | 1 | 1 | 3 | 12 |  | **2.5** | **17** |  | 3 | 4 | 4 | 19 |  | **2.3** | **13** |
| 67 |  |  |  | 1 | 12 |  | **2.9** | **13** |  |  |  | 1 | 16 |  | **2.9** | **17** |  | 0 | 0 | 2 | 28 |  | **2.9** | **13** |
| 86 |  |  | 2 | 2 | 9 |  | **2.5** | **13** |  | 1 | 3 | 5 | 8 |  | **2.2** | **17** |  | 1 | 5 | 7 | 17 |  | **2.3** | **13** |
| 93 |  |  | 3 | 1 | 9 |  | **2.5** | **13** |  |  | 5 | 1 | 11 |  | **2.4** | **17** |  | 0 | 8 | 2 | 20 |  | **2.4** | **13** |
| 100 |  |  | 1 | 2 | 10 |  | **2.7** | **13** |  | 1 | 2 | 5 | 9 |  | **2.3** | **17** |  | 1 | 3 | 7 | 19 |  | **2.5** | **13** |
| 105 |  |  | 1 | 5 | 7 |  | **2.5** | **13** |  |  | 5 | 8 | 4 |  | **1.9** | **17** |  | 0 | 6 | 13 | 11 |  | **2.2** | **13** |
| 110 |  | 2 | 2 | 5 | 4 |  | **1.8** | **13** |  | 3 | 4 | 7 | 3 |  | **1.6** | **17** |  | 5 | 6 | 12 | 7 |  | **1.7** | **13** |
| 111 |  | 1 |  | 4 | 8 |  | **2.5** | **13** |  | 1 | 5 | 5 | 6 |  | **1.9** | **17** |  | 2 | 5 | 9 | 14 |  | **2.2** | **13** |
| 114 |  |  |  | 5 | 8 |  | **2.6** | **13** |  |  |  | 4 | 10 |  | **2.7** | **14** |  | 0 | 0 | 9 | 18 |  | **2.4** | **13** |
| 1 |  |  | 1 | 7 | 4 |  | **2.3** | **12** |  | 4 | 3 | 5 | 6 |  | **1.7** | **18** |  | 4 | 4 | 12 | 10 |  | **1.9** | **12** |
| 2 |  |  | 1 | 5 | 6 |  | **2.4** | **12** |  | 1 | 1 | 6 | 9 |  | **2.4** | **17** |  | 1 | 2 | 11 | 15 |  | **2.3** | **12** |
| 30 |  |  |  | 1 | 11 |  | **2.9** | **12** |  |  | 4 | 7 | 7 |  | **2.2** | **18** |  | 0 | 4 | 8 | 18 |  | **2.5** | **12** |
| 76 |  |  | 3 | 3 | 6 |  | **2.3** | **12** |  |  | 4 | 6 | 6 |  | **2.1** | **16** |  | 0 | 7 | 9 | 12 |  | **2.0** | **12** |
| 80 |  |  |  | 5 | 7 |  | **2.6** | **12** |  | 1 | 1 | 6 | 7 |  | **2.3** | **15** |  | 1 | 1 | 11 | 14 |  | **2.2** | **12** |
| 97 |  |  |  | 5 | 7 |  | **2.6** | **12** |  | 2 | 5 | 6 | 5 |  | **1.8** | **18** |  | 2 | 5 | 11 | 12 |  | **2.1** | **12** |
| 12 |  |  |  | 4 | 7 |  | **2.6** | **11** |  |  |  | 5 | 14 |  | **2.7** | **19** |  | 0 | 0 | 9 | 21 |  | **2.7** | **11** |
| 27 |  |  |  | 6 | 5 |  | **2.5** | **11** |  |  |  | 11 | 8 |  | **2.4** | **19** |  | 0 | 0 | 17 | 13 |  | **2.4** | **11** |
| 37 |  |  | 1 | 1 | 9 |  | **2.7** | **11** |  | 1 | 5 | 1 | 11 |  | **2.2** | **18** |  | 1 | 6 | 2 | 20 |  | **2.3** | **11** |
| 42 |  |  | 1 | 1 | 9 |  | **2.7** | **11** |  |  | 3 | 3 | 13 |  | **2.5** | **19** |  | 0 | 4 | 4 | 22 |  | **2.6** | **11** |
| 88 |  | 1 |  | 1 | 9 |  | **2.6** | **11** |  | 3 | 4 | 2 | 10 |  | **2.0** | **19** |  | 4 | 4 | 3 | 19 |  | **2.2** | **11** |
| 108 |  | 1 | 1 | 2 | 7 |  | **2.4** | **11** |  | 5 | 2 | 5 | 7 |  | **1.7** | **19** |  | 6 | 3 | 7 | 14 |  | **2.0** | **11** |
| 40 |  | 1 |  | 8 | 1 |  | **1.9** | **10** |  | 1 | 5 | 12 | 1 |  | **1.7** | **19** |  | 2 | 5 | 20 | 2 |  | **1.7** | **10** |
| 50 |  |  | 1 | 2 | 7 |  | **2.6** | **10** |  |  | 4 | 6 | 8 |  | **2.2** | **18** |  | 0 | 5 | 8 | 15 |  | **2.2** | **10** |
| 91 |  |  | 3 | 3 | 4 |  | **2.1** | **10** |  | 1 | 5 | 7 | 7 |  | **2.0** | **20** |  | 1 | 8 | 10 | 11 |  | **2.0** | **10** |
| 4 |  | 1 | 4 | 4 |  |  | **1.3** | **9** |  | 3 | 5 | 4 | 4 |  | **1.6** | **16** |  | 4 | 9 | 8 | 4 |  | **1.2** | **9** |
| 34 |  | 1 | 3 | 2 | 3 |  | **1.8** | **9** |  | 5 | 4 | 6 | 4 |  | **1.5** | **19** |  | 6 | 7 | 8 | 7 |  | **1.5** | **9** |
| 57 |  |  |  |  | 9 |  | **3.0** | **9** |  |  | 3 | 7 | 11 |  | **2.4** | **21** |  | 0 | 3 | 7 | 20 |  | **2.6** | **9** |
| 61 |  |  |  | 2 | 7 |  | **2.8** | **9** |  |  | 5 | 8 | 7 |  | **2.1** | **20** |  | 0 | 5 | 10 | 14 |  | **2.2** | **9** |
| 66 |  |  | 2 |  | 7 |  | **2.6** | **9** |  | 2 | 2 | 8 | 7 |  | **2.1** | **19** |  | 2 | 4 | 8 | 14 |  | **2.1** | **9** |
| 5 |  | 1 |  | 1 | 6 |  | **2.5** | **8** |  |  |  | 6 | 16 |  | **2.7** | **22** |  | 1 | 0 | 7 | 22 |  | **2.7** | **8** |
| 17 |  |  |  | 3 | 5 |  | **2.6** | **8** |  | 2 |  | 5 | 15 |  | **2.5** | **22** |  | 2 | 0 | 8 | 20 |  | **2.5** | **8** |
| 33 |  |  |  | 4 | 4 |  | **2.5** | **8** |  | 2 | 3 | 8 | 9 |  | **2.1** | **22** |  | 2 | 3 | 12 | 13 |  | **2.2** | **8** |
| 45 |  |  |  | 4 | 4 |  | **2.5** | **8** |  | 1 | 7 | 11 | 2 |  | **1.7** | **21** |  | 1 | 7 | 15 | 6 |  | **1.8** | **8** |
| 47 |  |  |  | 1 | 7 |  | **2.9** | **8** |  |  | 1 | 9 | 12 |  | **2.5** | **22** |  | 0 | 1 | 10 | 19 |  | **2.6** | **8** |
| 81 |  |  | 2 | 6 |  |  | **1.8** | **8** |  |  | 5 | 15 | 1 |  | **1.8** | **21** |  | 0 | 7 | 21 | 1 |  | **1.7** | **8** |
| 52 |  |  |  | 3 | 4 |  | **2.6** | **7** |  |  | 1 | 14 | 7 |  | **2.3** | **22** |  | 0 | 1 | 17 | 11 |  | **2.3** | **7** |
| 16 |  | 3 |  | 2 | 1 |  | **1.2** | **6** |  | 6 | 4 |  | 3 |  | **1.0** | **13** |  | 9 | 4 | 2 | 4 |  | **0.7** | **6** |
| 55 |  |  |  | 1 | 5 |  | **2.8** | **6** |  |  | 1 | 1 | 19 |  | **2.9** | **21** |  | 0 | 1 | 2 | 24 |  | **2.6** | **6** |
| 89 |  |  | 2 | 3 | 1 |  | **1.8** | **6** |  | 1 | 5 | 8 | 1 |  | **1.6** | **15** |  | 1 | 7 | 11 | 2 |  | **1.2** | **6** |
| 62 |  |  | 1 |  | 4 |  | **2.6** | **5** |  | 3 | 2 | 5 | 15 |  | **2.3** | **25** |  | 3 | 3 | 5 | 19 |  | **2.3** | **5** |
| 113 |  | 2 | 1 |  | 2 |  | **1.4** | **5** |  | 3 | 8 | 4 | 2 |  | **1.3** | **17** |  | 5 | 9 | 4 | 4 |  | **1.0** | **5** |